Testing, monitoring, and dating structural changes in exchange rate regimes

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ARTICLE INFO	ABSTRACT
	Linear regression models for *de facto* exchange rate regime classification are complemented by inferential techniques for evaluating the stability of the regimes. To simultaneously assess parameter instabilities in the regression coefficients and the error variance an (approximately) normal regression model is adopted and a unified toolbox for testing, monitoring, and dating structural changes is provided for general (quasi-)likelihood-based regression models. Subsequently, the toolbox is employed for investigating the Chinese exchange rate regime after China gave up on a fixed exchange rate to the US dollar in 2005 and for tracking the evolution of the Indian exchange rate regime from 1993 until 2008. © 2009 Elsevier B.V. All rights reserved.

1. Introduction

The exchange rate regime of a country determines how it manages its currency with respect to foreign currencies: e.g., fixed covertability, floating based on market forces, or a so-called "pegged" regime in between these two extremes. As many central banks do not reveal full information about their exchange rate regime, researchers and practitioners typically track the evolution of the regime in operation using regression models based on rolling or shifting data windows. However, such a strategy lacks a formal inferential framework for determining changes in the regimes. In this paper, we provide such a framework by adapting structural change tools from the statistics and econometrics literature to the specific challenges of exchange rate regressions. This is accomplished by first establishing a unified structural change toolbox for general (quasi-)likelihood-based regressions and then employing an (approximately) normal linear regression model to capture parameter instabilities in both regression coefficients and error variance.

The analysis of exchange rate regimes received increasing interest during the last decade when it became clear that the *de jure* exchange rate regime in a country, as announced by the central bank, often differs from the *de facto* regime in operation (see e.g., Reinhart and Rogoff, 2004; Levy-Yeyati and Sturzenegger, 2003; Bubula and Ötker-Robe, 2002). A valuable tool for understanding the de facto exchange rate regime in operation is a linear regression model based on cross-currency exchange rates (with respect to a suitable numeraire) which has been used at least since Haldane and Hall (1991) and has been popularized by Frankel and Wei (1994). As the timing of interventions in the exchange rate regime by the corresponding central bank is often unknown, it is typically unclear what is the best data window for fitting an exchange rate regression. More precisely, after estimating a model the following questions are of interest:

1. Is a given exchange rate model stable within the time period in which it was established?
2. If it is stable, does it remain stable for future incoming observations?
3. If it is not stable, when and how did the exchange rate regime change?

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Here, we embed these questions into a structural change framework where the first question is referred to as *testing* for structural change (see e.g., Hansen, 2001; Zeileis, 2005), the second as *monitoring* structural change (see e.g., Chu et al., 1996; Zeileis et al., 2005), and the third as estimating breakpoints, also known as *dating* structural changes (see e.g., Bai and Perron, 2003). These techniques are well established for inference about the coefficients in least-squares regressions (see e.g., Zeileis et al., 2003, for a practical overview). However, for the exchange rate regression, changes in the error variance are of prime interest. Therefore, we extend all techniques to incorporate the error variance as a full model parameter by adopting an (approximately) normal model. More generally, we provide a unifying view that illustrates how a functional central limit theorem for a (quasi-)likelihood-based model can be employed for testing, monitoring, and dating in this model. Specifically, this extends the dating algorithm for ordinary-least-squares (OLS) regression (Bai and Perron, 2003) to (quasi-)maximum likelihood (ML and QML) models.

Subsequently, the suggested techniques are applied to investigate the exchange rate regimes of two currencies: First, we assess the evolution of China's exchange rate regime after abandoning a fixed exchange rate between the Chinese yuan CNY and the US dollar USD (People's Bank of China, 2005). Second, the number and structure of exchange rate regimes for the Indian rupee INR since April 1993 is analyzed.

2. Exchange rate regime analysis

In the last decade, it has been revealed that the de jure exchange rate regime in a country, as announced by the central bank, often differs from the de facto regime in operation. This has motivated a literature on data-driven methods for the classification of exchange rate regimes (see e.g., Reinhart and Rogoff, 2004; Levy-Yeyati and Sturzenegger, 2003; Bubula and Ötker-Robe, 2002). It has been attempted to create datasets identifying the exchange rate regime in operation for all countries in recent decades, using a variety of alternative algorithms. While these classification schemes are widely used, the algorithms involve numerous ad hoc constants, and are relatively weak on their statistical foundations.

Broadly speaking, exchange rate regimes range from *floating* (i.e., the currency is allowed to fluctuate based on market forces) over *pegged* (i.e., the currency has limited flexibility when compared with a basket of currencies or a single currency) to *fixed* (i.e., the currency has a fixed parity to another currency). To determine whether a certain currency is pegged to (a basket of) other currencies, there is a standard regression model popularized by Frankel and Wei (1994), based on returns of cross-currency exchange rates (with respect to a suitable numeraire). Recent applications of this model include Bénassy-Quéré et al. (2006), Shah et al. (2005) and Frankel and Wei (2007). More precisely, the exchange rate model is a standard linear regression

$$y_i = x_i^\top \beta + u_i \quad (i = 1, \ldots, n), \tag{1}$$

in which the y_i are *returns* of the target currency and the x_i are vectors of *returns* for a basket of c currencies plus a constant. When a country runs a fixed exchange rate, one element of β is 1 and the remaining elements are zero, and the error variance is $\sigma^2 = 0$. When a country runs a pegged exchange rate against one currency, one element of β is near 1, the remaining elements are near zero, and σ^2 takes low values. With a basket peg, σ^2 takes low values, and the coefficients β correspond to weights of the basket. With a floating rate, σ^2 is high, and the β values reflect the natural current account and capital account linkages of the country. Consequently, the error variance σ^2, or somewhat more intuitively the associated R^2 value, reflects the amount of pegging. R^2 values in excess of 99% are not unusual for tightly pegged currencies while much lower values are obtained for floating currencies.

For both y_i and x_i in Eq. (1), log-difference returns (in percent) of different currencies are typically used — as computed by $100 \cdot (\log p_i - \log p_{i-1})$, where p_i is the price of a currency at time i in some numeraire currency. In the applications below, we always employ CHF (Swiss franc) as the numeraire currency. For the numeraire, other choices are conceivable (e.g., other currencies, special drawing rights, or gold); but for pegged exchange rate regimes the results are typically not very sensitive to the choice of the numeraire (Frankel and Wei, 1994, 2007). More generally, exchange rate regressions for pegged regimes are typically exceptionally well behaved: regressors are stationary (due to taking log-difference returns), R^2 is high, autocorrelation is low (because central banks mostly react to changes in the reference currencies rather than changes in their own currency). However, an obstacle in establishing an exchange rate regression is that it is often not known if and when shifts/breaks in the regime occur. Note that breaks (rather than smooth transitions) are particularly likely to be a useful model here because changes in the exchange rate regime typically stem from policy interventions of the corresponding central banks.

Assessing the stability of an exchange rate regression might seem trivial because it is a linear regression model, typically estimated by OLS, for which application of all structural change techniques is well-established practice. However, the error variance σ^2 (capturing the flexibility of the exchange rate regime in operation) is of crucial interest here and has to be treated as a full model parameter and not just as a nuisance parameter as in most linear regressions and associated structural change methods. Specifically, RSS-based structural change techniques (such as Bai and Perron, 2003; Zeileis et al., 2003) are insensitive to changes in σ^2, if it is not included explicitly. A straightforward alternative is to adopt a quasi-normal model with density

$$f(y \mid x, \beta, \sigma^2) = \phi((y - x^\top \beta)/\sigma)/\sigma, \tag{2}$$

where $\phi(\cdot)$ is the standard normal density function. This has the full combined parameter $\theta = (\beta^\top, \sigma^2)^\top$ of length $k = c + 2$

(c currency coefficients, intercept, and variance). For this model, QML and OLS lead to the same estimates for the coefficients β. However, by adding the error variance σ^2 as a full model parameter, it is easily included in the inference and parameter stability can be assessed jointly for β and σ^2 (see e.g., Hansen, 1992; Perron and Zhou, 2008). Unfortunately, the structural change methods for ML/QML models are somewhat scattered in the literature and no unified framework is readily available. Hence, in the next section, we collect suitable testing and monitoring methods and extend the dating procedure of Bai and Perron (2003) to ML models.

3. A unified structural change toolbox for ML models

In this section, we first outline the model frame for regression models estimated by ML/QML along with the usual central limit theorem (CLT). For structural change methods, this CLT needs to be extended to a functional CLT (FCLT) based on which testing procedures (in historical samples), monitoring procedures (sequential tests for incoming data), and confidence intervals for breakpoints can be established, yielding a unified structural change toolbox for ML/QML models. Based on this, the adaptation of the exchange rate regression from Eqs. (1) and (2) is applied in Section 4 to two currencies.

3.1. Model

We assume n observations of some dependent variable y_i and a regressor vector x_i, such that the conditional distribution $y_i \mid x_i$ follows some (quasi-)likelihood $f(y_i \mid x_i, \theta_i)$ with k-dimensional parameter θ_i. The ordering of the observations usually corresponds to time. Then, the hypothesis of interest is "parameter stability", i.e.,

$$H_0 : \theta_i = \theta_0 \quad (i = 1, \ldots, n), \tag{3}$$

that should be tested against the alternative that the parameter θ_i changes over time.

If the parameters θ in such a model are stable, they can be estimated by minimizing the corresponding negative log-likelihood (NLL) $\Psi_{\text{NLL}}(y_i, x_i, \theta) = -\log f(y_i \mid x_i, \theta)$ yielding the QML estimate $\hat{\theta}$. Equivalently – instead of minimizing the objective function $\Psi(y_i, x_i, \theta)$ – the corresponding first-order conditions can be solved based on the associated estimating functions $\psi(y_i, x_i, \theta) = \partial \Psi(y_i, x_i, \theta)/\partial \theta$:

$$\underset{\theta \in \Theta}{\operatorname{argmin}} \sum_{i=1}^{n} \Psi(y_i, x_i, \theta) = \hat{\theta}, \tag{4}$$

$$\sum_{i=1}^{n} \psi(y_i, x_i, \hat{\theta}) = 0. \tag{5}$$

We adopt the NLL Ψ_{NLL} as the objective function Ψ throughout the paper but note that many of the methods discussed below can be applied straightforwardly to other M-type estimators such as ordinary least squares (OLS), nonlinear least squares (NLS), or robust M estimators.

Under the assumption that the parameters are stable and can be consistently estimated, and given standard regularity conditions, e.g., as in White (1994, Theorem 6.10, p. 104) or Cameron and Trivedi (2005, Chapter 5), a central limit theorem (CLT) holds:

$$\sqrt{n}(\hat{\theta} - \theta_0) \xrightarrow{\text{d}} \mathcal{N}(0, A_0^{-1} B_0 A_0^{-1}), \tag{6}$$

i.e., $\hat{\theta}$ is asymptotically normal with mean θ_0 and a sandwich-type covariance matrix with components

$$A_0 = \operatorname{plim} n^{-1} \sum_{i=1}^{n} \mathsf{E}[-\psi'(y_i, x_i, \theta_0)], \tag{7}$$

$$B_0 = \operatorname{plim} n^{-1} \sum_{i=1}^{n} \mathsf{VAR}[\psi(y_i, x_i, \theta_0)], \tag{8}$$

where ψ' is the derivative of ψ, again with respect to θ.

Under various sets of assumptions (see e.g., Andrews, 1993), this CLT can be extended to a functional CLT (FCLT): The empirical fluctuation process $efp(\cdot)$, defined as the decorrelated partial sum process of the empirical estimating functions, converges to a k-dimensional Brownian bridge $W^0(\cdot)$ on the interval [0, 1] (see e.g., Zeileis, 2005).

$$efp(t) = \hat{B}^{-1/2} n^{-1/2} \sum_{i=1}^{\lfloor nt \rfloor} \psi(y_i, x_i, \hat{\theta}) \quad (0 \le t \le 1), \tag{9}$$

$$efp(\cdot) \xrightarrow{\text{d}} W^0(\cdot). \tag{10}$$

The Brownian bridge can also be written as $W^0(t) = W(t) - tW(1)$, where $W(\cdot)$ is a standard k-dimensional Brownian motion. Furthermore, $\hat{B}$ is some suitable estimator of B_0, e.g., the outer product of the empirical estimating functions or

some HC (heteroskedasticity consistent) or HAC (heteroskedasticity and autocorrelation consistent) estimator to allow for certain deviations from the correct specification of the likelihood.

While the empirical fluctuation process $efp(\cdot)$ is governed by the FCLT under the null hypothesis of parameter stability, its fluctuation should generally be increased under the alternative of structural change. In particular, the process typically exhibits peaks at the times where changes in θ_i occur. Hence, $efp(\cdot)$ and the associated FCLT are the basis for the inference presented in the remainder of this section.

3.2. Testing

The classical question in structural change analysis is whether the model parameter θ is really stable throughout the sample period $i = 1, \ldots, n$. Thus, testing for structural change is concerned with testing the null hypothesis H_0 from Eq. (3) against the alternative that θ_i changes over time. Various patterns of change are conceivable, e.g., single or multiple breaks or random walks etc. Many different types of tests have been suggested in the econometrics and statistics literature for this problem, directing power against different types of departure from H_0. Zeileis (2005) provides a unifying view on a wide collection of methods, using a general class of so-called M-fluctuation tests that can be directly applied to ML and QML models. Hence, for brevity, we just outline the class of M-fluctuation tests and refer to Zeileis (2005) for more details. Applications to several likelihood-based regression models (Poisson, binomial, and beta regression) can be found in Zeileis (2006).

The basic idea of M-fluctuation tests is that deviations from parameter stability can be brought out by assessing deviations of the empirical estimating functions $\psi(y_i, x_i, \hat{\theta})$ from their zero mean. This is done by applying some aggregation functional $\lambda(\cdot)$ to the empirical fluctuation process $\lambda(efp)$, yielding a univariate test statistic. The corresponding limiting distribution is the same functional (or its asymptotic counterpart) applied to a Brownian bridge $\lambda(W^0)$ so that critical values and p values can be derived. Important special cases include:

$$S_{dmax} = \sup_{t \in [0,1]} \|efp(t)\|_\infty, \tag{11}$$

$$S_{CvM} = n^{-1} \sum_{i=1}^{n} \|efp(i/n)\|_2^2, \tag{12}$$

$$S_{MOSUM} = \sup_{t \in [0,1-h]} \|efp(t+h) - efp(t)\|_\infty, \tag{13}$$

$$S_{\sup LM} = \sup_{t \in [\pi, 1-\pi]} \frac{\|efp(t)\|_2^2}{t(1-t)}. \tag{14}$$

Selection of some optimal statistic is an inherently difficult problem for structural change tests due to the vast alternative of parameter instability (unless the exact pattern of deviation from stability were known). However, a few guidelines can be established that facilitate selection of an appropriate testing procedure for typical situations occurring practice: The double maximum statistic S_{dmax} (i.e., the maximum over the k components and time t) is particularly useful for exploratory purposes because it can be easily visualized along with critical values (derived from the distribution of the maximum of a Brownian bridge) so that timing of a structural change and the parameter affected by it can be identified graphically. However, this test might have poor power in the presence of a random walk alternative or multiple breaks. In such a situation, a Cramér–von Mises statistic S_{CvM} as in the Nyblom–Hansen test (Nyblom, 1989; Hansen, 1992) or a MOSUM (moving sum) statistic S_{MOSUM} with bandwidth h would be more suitable. The statistic $S_{\sup LM}$ (Andrews, 1993) is the supremum of LM (Lagrange multiplier) statistics on the interval $[\pi, 1-\pi]$ (for some trimming parameter π) and is particularly well suited for single break alternatives. Moreover, test statistics employing the L_2 norm for aggregating over the components of the fluctuation process (such as S_{CvM} and $S_{\sup LM}$) will perform better if several (or even all) components of the parameter vector change at the same time. On the other hand, if only one of many components in the parameter vector is subject to a structural change, then statistics based on the L_∞ norm (such as S_{dmax} and S_{MOSUM}) will typically have higher power.

3.3. Monitoring

Given that a stable model could be established for observations $i = 1, \ldots, n$, it is natural to ask whether this model remains stable for future incoming observations $i > n$. More formally, based on the assumption that Eq. (3) holds, monitoring (Chu et al., 1996) tests the null hypothesis

$$H_0 : \theta_i = \theta_0 \quad (i > n), \tag{15}$$

sequentially against changes in the so-called monitoring period $i > n$ (or the scaled $t > 1$).

Based on the tools from the previous section, an extension to the monitoring situation is fairly straightforward. The empirical fluctuation process $efp(t)$ is simply continued in the monitoring period by computing the empirical estimating functions for each new observation (using the parameter estimates from the stable history period $[0, 1]$) and updating the

cumulative sum process. This is still governed by an FCLT on an extended interval $[0, T]$ with $T > 1$ (Zeileis, 2005). Based on this FCLT, a testing procedure can be established that re-computes the functional $\lambda(efp(t))$ for each new observation and rejects the null hypothesis from Eq. (15) if it exceeds some critical value $b(t)$ for any $t > 1$. As this is a sequential testing procedure, not only a single critical value is required but a function $b(t)$ that can be interpreted as a boundary function for the empirical fluctuation process. To yield a level α testing procedure, it needs to fulfill

$$1 - \alpha = P(\lambda(W^0(t)) \leq b(t) \mid t \in [1, T]).$$

Various combinations of functionals λ and boundaries b are conceivable (Chu et al., 1996; Horváth et al., 2004; Zeileis et al., 2005; Zeileis, 2005) that can direct power against changes occurring early or late in the monitoring period $t > 1$ or that try to spread the power evenly.

For the applications in Section 4 we adopt a maximum functional and a linear boundary function $b(t) = \pm c \cdot t$ as suggested in Zeileis et al. (2005) that spreads the power rather evenly. More precisely, we detect a change and reject the null hypothesis if

$$\|efp(t)\|_\infty > c \cdot t \quad \text{for any } t \in [1, T],$$

where the critical value c can be obtained from Zeileis et al. (2005, Table III) using a Bonferroni correction.

3.4. Dating

If there is evidence for parameter instability in the regression model, a natural question is to ask when and how the parameters changed. Often, a reasonable approximation is to adopt a segmented regression model, i.e., assume stable sets of parameters $\theta^{(j)}$ exist for $j = 1, \ldots, m + 1$ segments that are mutually exclusive and cover the sample period. More formally, $\theta^{(j)}$ holds for the observations $i = i_{j-1} + 1, \ldots, i_j$ where $\{i_1, \ldots, i_m\}$ are the m breakpoints and, by convention, $i_0 = 0$ and $i_{m+1} = n$.

The goal of dating is to determine estimates of the m breakpoints and the $m + 1$ segment-specific parameters $\theta^{(j)}$, often followed by a subsequent model selection for the number of breakpoints m. If the breakpoints were known, estimation of the parameters $\theta^{(j)}$ would be easy: they can be obtained by solving Eq. (4) (or (5)) in the jth segment. Hence, the overall segmented objective function based on Ψ is given by

$$PSI(i_1, \ldots, i_m) = \sum_{j=1}^{m+1} psi(i_{j-1} + 1, i_j),$$

$$psi(i_{j-1} + 1, i_j) = \sum_{i=i_{j-1}+1}^{i_j} \Psi(y_i, x_i, \hat{\theta}^{(j)}),$$

where $psi(i_{j-1} + 1, i_j)$ is the minimal value of the objective function for the model fitted on the jth segment with associated parameter estimate $\hat{\theta}^{(j)}$. Dating then tries to find the global optimizers $\hat{i}_1, \ldots, \hat{i}_m$ of the segmented objective function, i.e., solving

$$(\hat{i}_1, \ldots, \hat{i}_m) = \underset{(i_1, \ldots, i_m)}{\operatorname{argmin}} PSI(i_1, \ldots, i_m), \tag{16}$$

subject to a minimal segment size constraint $i_j - i_{j-1} + 1 \geq n_h \geq k$. The minimal segment size is either chosen directly or derived from some trimming h as $n_h = \lfloor nh \rfloor$. The optimal (with respect to Ψ) set of breakpoints from Eq. (16) is called m-partition $\mathcal{I}_{m,n} = \{\hat{i}_1, \ldots, \hat{i}_m\}$.

Estimation

Direct optimization in Eq. (16) by exhaustive search over all conceivable partitions is of order $O(n^m)$ and hence computationally burdensome. Fortunately, the Bellman principle of optimality can be applied to the problem as the following recursion holds:

$$PSI(\mathcal{I}_{m,n}) = \min_{mn_h \leq i \leq n-n_h} \left[PSI(\mathcal{I}_{m-1,i}) + psi(i + 1, n) \right]. \tag{17}$$

Therefore, a dynamic programming approach can be employed that solves the global minimization in $O(n^2)$. Bai and Perron (2003) describe in detail such a dynamic programming algorithm for minimizing the segmented residual sum of squares (RSS) in linear regression models. In fact, the same algorithm can be applied for computing the more general class of estimators considered here because the objective function is additive in the observations (Hawkins, 2001). More precisely, for computing the QML estimators in some quasi-likelihood-based model (instead of OLS estimators in the linear regression model) their objective function Ψ_{RSS} just has to be replaced by the negative log-likelihood Ψ_{NLL}:

$$\Psi_{RSS}(\theta) = (y_i - x_i^\top \theta)^2,$$
$$\Psi_{NLL}(\theta) = -\log f(y_i \mid x_i, \theta).$$

Essentially, the algorithm first computes a triangular matrix with $psi(i, j)$ for all $j - i \geq \lfloor nh \rfloor$ and $i = 1, \ldots, n - \lfloor nh \rfloor + 1$. Based on this matrix, the problem from Eq. (16) can be solved by exploiting Eq. (17) for any number of breakpoints m (if $(m + 1)\lfloor nh \rfloor < n$).

Confidence intervals

In addition to point estimation, confidence intervals for the true segment-specific parameters $\theta_0^{(j)}$ and the true break-points i_j^0 are of interest. Hence, we suggest an extension of the results of Bai and Perron (2003) to QML models. As the breakpoint estimates $\hat{i}_j$ converge at the faster rate n, the standard $\sqrt{n}$ asymptotics from Eq. (6) still hold for $\hat{\theta}^{(j)}$ with segment-specific matrices $A_0^{(j)}$ and $B_0^{(j)}$ (analogous to Eqs. (7) and (8)), respectively. Both can be estimated in the usual way, e.g., by computing a HC or HAC estimate from the observations in segment j. If the likelihood can be assumed to be correctly spec-ified, then $A_0^{(j)} = B_0^{(j)}$ corresponds to the Fisher information and is usually estimated by the Hessian matrix.

Confidence intervals for the true breakpoints i_j^0 can also be derived if, following Bai and Perron (2003), an asymptotic framework is adopted where the magnitudes of the changes $\Delta_j = \theta_0^{(j+1)} - \theta_0^{(j)}$ converges to zero as the sample size increases. Then the distribution of the breakpoint estimates is given by

$$\frac{\Delta_j^\top A_0^{(j)} \Delta_j}{\Delta_j^\top B_0^{(j)} \Delta_j}(\hat{i}_j - i_j^0) \xrightarrow{\mathrm{d}} \operatorname*{argmax}_t V^{(j)}(t) \quad (j = 1, \ldots, m), \tag{18}$$

where $V^{(j)}(\cdot)$ is a stochastic process defined by

$$V^{(j)}(t) = \begin{cases} W_1^{(j)}(-t) - |t|/2, & \text{if } t \leq 0 \\ \sqrt{\xi_j}(\phi_2^{(j)}/\phi_1^{(j)})W_2^{(j)}(t) - \xi_j|t|/2, & \text{if } t > 0. \end{cases}$$

$W_1^{(j)}$ and $W_2^{(j)}$ are independent Brownian motions and $\xi_j = (\Delta_j^\top A_0^{(j+1)} \Delta_j)/(\Delta_j^\top A_0^{(j)} \Delta_j)$, $\phi_1^{(j)} = (\Delta_j^\top B_0^{(j)} \Delta_j)/(\Delta_j^\top A_0^{(j)} \Delta_j)$, and $\phi_2^{(j)} = (\Delta_j^\top B_0^{(j+1)} \Delta_j)/(\Delta_j^\top A_0^{(j+1)} \Delta_j)$. A closed form solution for the distribution function of $\operatorname{argmax}_t V^{(j)}(t)$ is provided by Bai (1997, Appendix B). Hence, all that is required in addition for computing confidence intervals are estimates $\hat{\Delta}_j = \hat{\theta}^{(j+1)} - \hat{\theta}^{(j)}$, as well as $\hat{A}^{(j)}$, and $\hat{B}^{(j)}$ which can be derived as above.

The steps for deriving Eq. (18) are completely analogous to Bai and Perron (2003, Section 4) and Bai (1997, Section II.C) if the role of regressors/disturbances in the linear regression model is replaced by the estimating functions in the QML model. Thus, the basic assumption is that an FCLT as in Eq. (10) has to hold for each segment j, corresponding to Bai (1997, Assumption 9). (The analogies to the OLS regression framework are easy to see when keeping in mind that the estimating functions for that model are given by $\psi(y_i, x_i, \theta) = x_i(y_i - x_i^\top \theta) = x_i u_i$ and the corresponding derivative is $\psi'(y_i, x_i, \theta) = x_i x_i^\top$.)

Model selection

Using the estimation algorithm described above, the optimal (with respect to Ψ) segmentation can be computed if the number of breakpoints m is known. In practice, however, m typically needs to be chosen based on the observed data as well. One solution to this problem is to compute the optimal segmentations for a sequence of breakpoints $m = 0, 1, \ldots$ (which can all be computed from the same triangular matrix mentioned above) and to choose m by optimizing some information criterion $IC(m)$. If the segmentations are likelihood-based, such information criteria are easily available. Thus, if $PSI(\ell_{m,n})$ is based on Ψ_{NLL} we call it $NLL(\ell_{m,n})$ and computation of information criteria is straightforward:

$$IC(m) = 2 \cdot NLL(\ell_{m,n}) + \text{penalty} \cdot ((m + 1)k + m),$$

with different penalties leading to different information criteria. Bai and Perron (2003) consider two different criteria, the BIC and a modified BIC as suggested by Liu et al. (1997):

$$\text{penalty}_{\mathrm{BIC}} = \log(n),$$
$$\text{penalty}_{\mathrm{LWZ}} = \alpha \cdot \log(n)^{2+\delta}.$$

In our empirical studies below, we follow the recommendations of Bai and Perron (2003) and Liu et al. (1997) and use the LWZ criterion, setting the parameters $\alpha = 0.299$ and $\delta = 0.1$ so that the LWZ penalty is higher than in the BIC for $n > 20$.

4. Applications

In this section, the methods presented above are applied to investigate the exchange rate regimes of China and India. The cross-currency returns are derived from exchange rates available online from the US Federal Reserve at http://www.federalreserve.gov/releases/h10/Hist/. All methods from Section 3 are based on the normal likelihood from Eq. (2) and the corresponding estimating functions, leading to suitable models on all segments determined by the dating procedure. On a few segments, some slight *negative* autocorrelation can be found which might indicate that the central banks try to balance

deviations into one direction by changes into the opposite direction in the next period. To account for this, inference could be based on HAC covariances (e.g., for $\hat{B}$ in Eq. (9)). However, as the autocorrelation is small in absolute size (if present at all), all results are almost unchanged and qualitatively identical when using standard covariances as done in the subsequent analyses.

All computations are carried out in the R system for statistical computing (R Development Core Team, 2009, version 2.10.0) with packages **fxregime** 1.0-0 (Zeileis et al., 2010) and **strucchange** 1.3-7 (Zeileis et al., 2002). R itself and the packages are freely available at no cost under the terms of the GNU General Public License (GPL) from the Comprehensive R Archive Network at http://CRAN.R-project.org/. Vignettes reproducing the analyses from this paper are available via `vignette("CNY", package = "fxregime")` and `vignette("INR", package = "fxregime")` after installing the packages.

4.1. China

In recent years, there has been enormous global interest in the CNY exchange rate which was fixed to the USD in the years leading up to mid-2005. In July 2005, China announced a small appreciation of CNY, and, in addition, a reform of the exchange rate regime. The People's Bank of China (PBC) announced this reform to involve a shift away from the fixed exchange rate to a basket of currencies with greater flexibility (People's Bank of China, 2005).

Despite the announcements of the PBC, little evidence could be found for China moving away from a USD peg in the months after July 2005 (Shah et al., 2005). To begin our investigation here, we follow up on our own analysis from autumn 2005: Using daily returns from 2005-07-26 up to 2005-10-31 (corresponding to $n = 68$), we established a stable exchange rate regression in Shah et al. (2005) that we monitored in the subsequent months, publishing the monitoring progress weekly online at http://www.mayin.org/ajayshah/papers/CNY_regime/. The currency basket employed consists of the most important floating currencies: USD, JPY (Japanese yen), EUR (euro), and GBP (British pound). More currencies could be included but we refrain from doing so because including irrelevant currencies decreases power and precision of the procedures and the most important conclusions can already be drawn with this small basket, as we will see below.

In a first step, we fit the exchange regression to the 68 observations in the first three months after the announcements of the PBC. The estimated exchange rate regime is

$$\text{CNY}_i = -0.005 + 0.9997\text{USD}_i + 0.005\text{JPY}_i - 0.014\text{EUR}_i - 0.008\text{GBP}_i + \hat{u}_i$$

where the ISO 4217 abbreviations denote currency returns computed from prices in CHF as described in Section 2. Only the USD coefficient differs significantly from 0 (but not significantly from 1), thus signalling a very clear USD peg, i.e., changes in USD are propagated into equal changes in CNY. The R^2 of the regression is 99.8% due to the extremely low standard deviation of $\sigma = 0.028$.

The fluctuation in the parameters during this history period is very small, see the corresponding $efp(t)$ for $0 \leq t \leq 1$ in Fig. 1 on the left of the vertical dashed line (marking the end of the history period). Also, none of the parameter instability tests from Section 3.2 would reject the null hypothesis of stability, e.g., the double maximum statistic is $S_{dmax} = 1.097$ ($p = 0.697$) or $S_{CvM} = 1.012$ ($p = 0.364$).

The same fluctuation process $efp(t)$ is continued subsequently as described in Section 3.3 in the monitoring period starting from 2005-11-01 as shown in Fig. 1 on the right of the vertical dashed line. The boundary shown is $b(t) = \pm 2.475 \cdot t$, derived at 5% significance level (for monitoring up to $T = 4$). In the first months, up to spring 2006, there is still moderate fluctuation in all processes signalling no departure from the previously established USD peg. In fact, the only larger deviation during that time period is surprisingly a *decrease* in the variance – corresponding to a somewhat tighter USD peg – which almost leads to a boundary crossing in January 2006. However, the situation relaxes somewhat before in March 2006 the variance component of the fluctuation process starts to deviate clearly from its zero mean. This corresponds to an increase in the variance and leads to a boundary crossing on 2006-03-27. The fluctuation in all other coefficients remains non-significant which conveys that a USD peg is still in operation, only with a somewhat larger variance.

To capture the changes in China's exchange rate regime more formally, we fit a segmented exchange rate regression by dating regime changes as described in Section 3.4. Using daily returns from 2005-07-26 through to 2009-07-31, we determine the optimal breakpoints for $m = 1, \ldots, 10$ breaks with a minimal segment size of $n_h = 20$ observations and compute the associated segmented NLL and LWZ criterion (see Fig. 2). Of course, NLL decreases with every additional break but with a particularly marked decrease for going from $m = 0$ to 1 break and smaller decreases for $m > 1$. This is also reflected in the LWZ criterion that clearly drops going from $m = 0$ to $m = 1$ and then assumes its minimum for $m = 3$ so that we choose a 3-break (or 4-segment) model. The estimated breakpoint associated with the pronounced drop in NLL and LWZ is 2006-03-14, i.e., shortly before the boundary crossing in the monitoring procedure (which occurs somewhat later due to the time the process needs to deviate from zero to the boundary). The two subsequent changes are 2008-08-22 and 2008-12-31. The corresponding parameter estimates are provided in Table 1 along with standard errors. The 90% confidence interval for the break dates estimate are [2006-02-21, 2006-03-15], [2008-07-31, 2008-08-25], and [2008-12-30, 2009-01-22] which are all rather non-symmetric. Roughly speaking, this means that for the first break the end of the low-variance regime can be determined more precisely than the start of the high-variance regime. Similar interpretations hold for the two other breaks.

These results allow for several conclusions about the Chinese exchange rate regime after July 2005: CNY was pegged to USD all the time. The exchange rate regime went from very tight in late 2005 and early 2006 ($R^2 = 99.8\%$) to somewhat
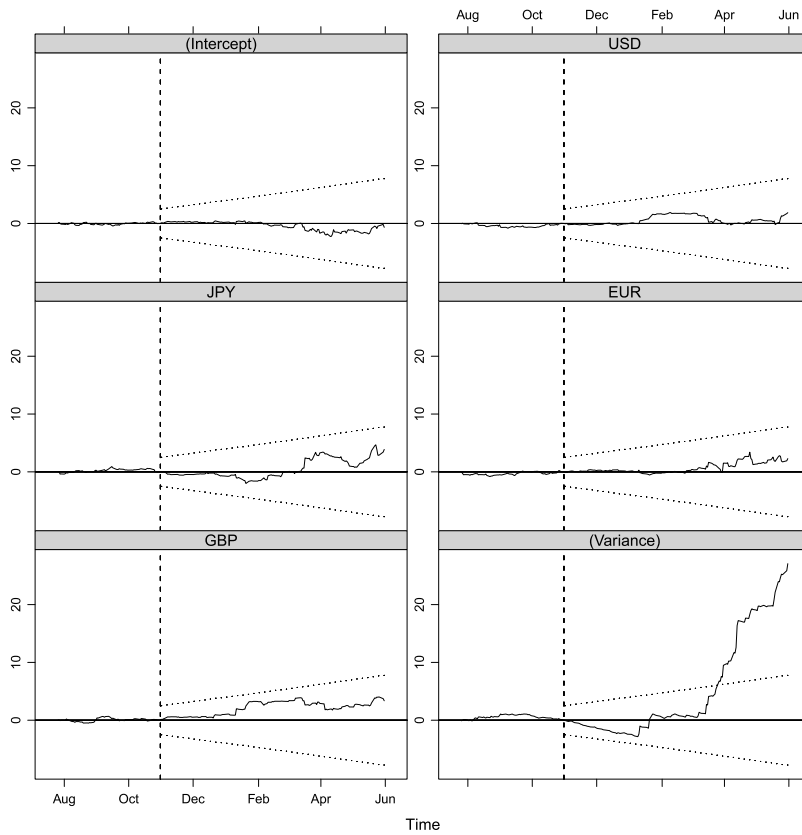
Fig. 1. Monitoring fluctuation process for the CNY exchange rate regime (from 2005-07-26 to 2006-05-31).
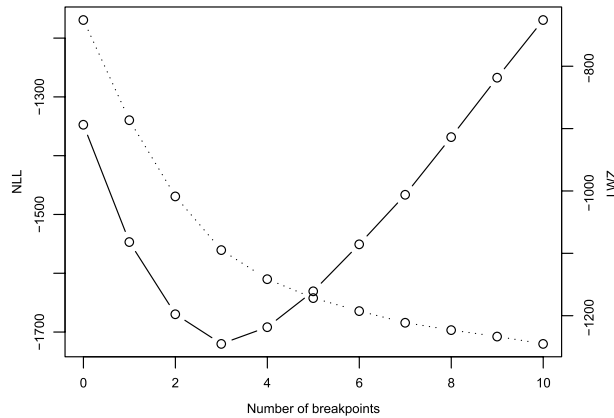

Fig. 2. Negative log-likelihood (dotted, left axis) and LWZ information criterion (solid, right axis) for CNY exchange rate regimes (from 2005-07-26 to 2009-07-31).

relaxed in 2006–2008 ($R^2 = 96.5\%$ and $R^2 = 95.6\%$, respectively) back to rather tight in 2009 ($R^2 = 99.8\%$). While the changes are statistically significant, variation is very low in all segments; for comparison, see the results below for India. Additionally to changes in the variance, the long second regime differs also with respect to the intercept. It is clearly smaller than 0, reflecting a slow appreciation of the CNY during that phase. In summary, there have only been modest relaxations of the initial rigid USD peg which do not reflect the original announcements of the PBC.

Table 1
Segmented CNY exchange rate regimes: Coefficient estimates (and standard errors) with significant coefficients (at 5% level) printed in bold face.

Start/end	β_0	β_{USD}	β_{JPY}	β_{EUR}	β_{GBP}	σ	R^2
2005-07-26	**−0.005**	**0.999**	0.005	−0.015	0.007	0.028	0.998
2006-03-14	(0.002)	(0.005)	(0.005)	(0.017)	(0.008)		
2006-03-15	**−0.025**	**0.969**	−0.009	0.026	−0.013	0.106	0.965
2008-08-22	(0.004)	(0.012)	(0.010)	(0.023)	(0.012)		
2008-08-25	−0.015	**1.031**	−0.026	0.049	0.007	0.263	0.956
2008-12-31	(0.030)	(0.044)	(0.030)	(0.059)	(0.035)		
2009-01-02	0.001	**0.981**	0.008	−0.008	0.009	0.044	0.998
2009-07-31	(0.004)	(0.005)	(0.004)	(0.009)	(0.004)		

4.2. India

Another expanding economy with a currency that has been receiving increased interest over the last years is India. As China, India is in the process of evolving away from a closed economy towards a greater integration with the world on both the current account and the capital account. This has brought considerable stress upon the pegged exchange rate regime.

Therefore, we try to track the evolution of the INR exchange rate regime since trading in the INR began. Using weekly returns from 1993-04-09 through to 2008-01-04 (yielding $n = 770$ observations), we first fit a single exchange rate regime that is subsequently segmented. Weekly rather than daily returns are employed to reduce the noise in the data and alleviate the computational burden of the dating algorithm of order $O(n^2)$. The currency basket employed is essentially the same as above using the most important floating currencies. (The only difference to the previous section is that EUR can only be used for the time after its introduction as official euro-zone currency in 1999. For the time before, exchange rates of the German mark – DEM, the most important currency in the EUR zone – adjusted to EUR rates, are employed. The combined returns are denoted DUR below.)

Using the full sample, we establish a single exchange rate regression only to show that there is not a single stable regime and to gain some exploratory insights from the associated fluctuation process. As we do not expect to be able to draw valid conclusions from the coefficients of a single regression, we do not report the coefficients here and rather move on to a
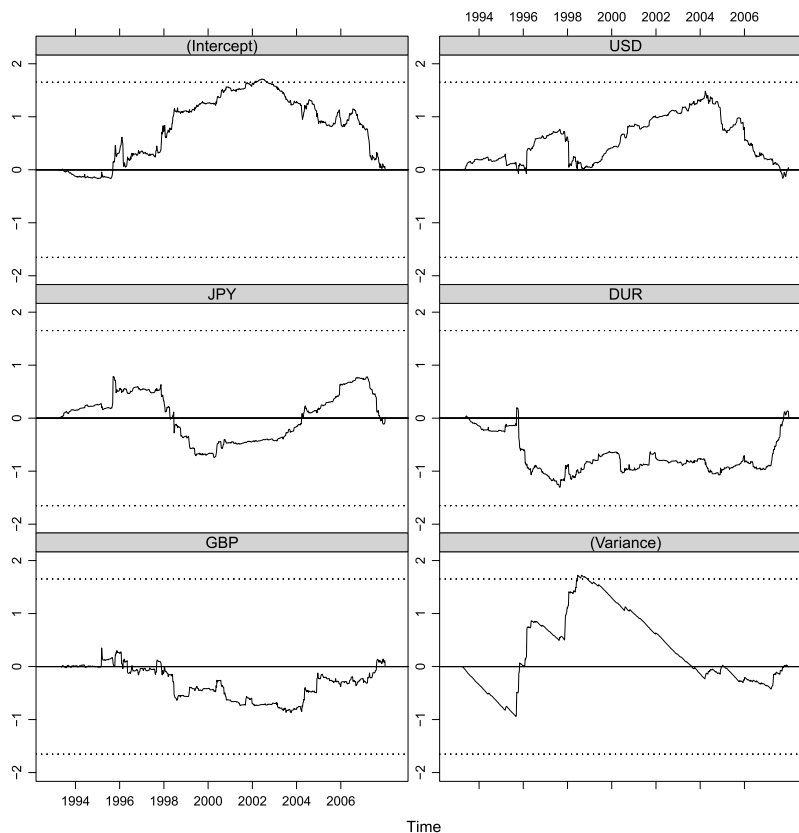

Fig. 3. Historical fluctuation process for the INR exchange rate regime (from 1993-04-02 to 2008-01-04).
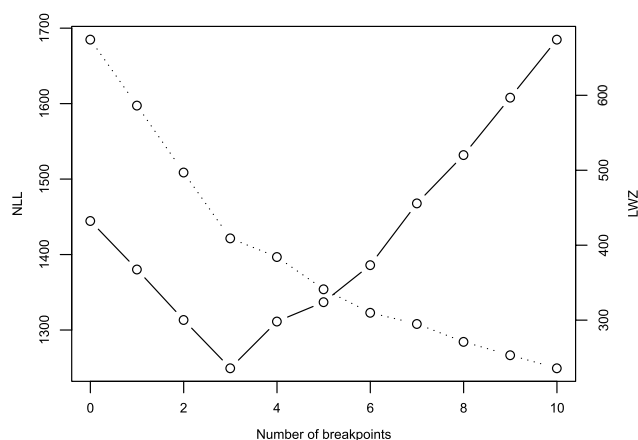
Fig. 4. Negative log-likelihood (dotted, left axis) and LWZ information criterion (solid, right axis) for INR exchange rate regimes (from 1993-04-09 to 2008-01-04).

Table 2
Segmented INR exchange rate regimes: Coefficient estimates (and standard errors) with significant coefficients (at 5% level) printed in bold face.

Start/end	β_0	β_{USD}	β_{JPY}	β_{DUR}	β_{GBP}	σ	R^2
1993-04-09	−0.006	**0.972**	0.023	0.011	0.020	0.157	0.989
1995-03-03	(0.017)	(0.018)	(0.014)	(0.032)	(0.024)		
1995-03-10	**0.161**	**0.943**	0.067	−0.026	0.042	0.924	0.729
1998-08-21	(0.071)	(0.074)	(0.048)	(0.155)	(0.080)		
1998-08-28	0.019	**0.993**	0.010	**0.098**	−0.003	0.275	0.969
2004-03-19	(0.016)	(0.016)	(0.010)	(0.034)	(0.021)		
2004-03-26	−0.058	**0.746**	**0.126**	**0.435**	**0.121**	0.579	0.800
2008-01-04	(0.042)	(0.045)	(0.042)	(0.116)	(0.056)		

visualization of $efp(t)$ and the associated double maximum test in Fig. 3. Because two processes (intercept and variance) exceed their 5% level boundaries, there is evidence for at least one structural change. More formally, the test statistic is $S_{dmax} = 1.724$ with a p value of $p = 0.031$. This p value is not very small because there seem to be several changes in various parameters. A more suitable test in such a situation would be the Nyblom–Hansen test with $S_{CvM} = 3.115$ and $p < 0.005$. Nevertheless, the multivariate fluctuation process is interesting as a visualization of the changes in the different parameters. The process for the variance σ^2 has the most distinctive shape revealing at least four different regimes: at first, a variance that is lower than the overall average (and hence a decreasing process), then a much larger variance (up to the boundary crossing), a much smaller variance again and finally a period where the variance is roughly the full-sample average. Other interesting processes are the intercept and maybe the USD and DUR. The latter two are not significant but have some peaks revealing a decrease and increase, respectively, in the corresponding coefficients.

To capture this exploratory assessment in a formal way, a dating procedure is conducted for $m = 1, \ldots, 10$ breaks and a minimal segment size of $n_h = 20$ observations. The resulting values for the NLL and associated LWZ information criterion are depicted in Fig. 4. NLL is decreasing quickly up to $m = 3$ breaks with a kink in the slope afterwards. Similarly, LWZ takes its minimum for $m = 3$ breaks, choosing a 4-segment model. The corresponding parameter estimates (and standard errors) are reported in Table 2.

The most striking observation from the segmented coefficients is that INR was closely pegged to USD up to March 2004 when it shifted to a basket peg in which USD has still the highest weight but considerably less than before. Furthermore, the changes in σ and R^2 are remarkable, roughly matching the exploratory observations from the empirical fluctuation process. A more detailed look at the results in Table 2 shows that the first period is a clear and tight USD peg. During that time, pressure to appreciate was blocked by purchases of USD by the central bank. The second period, including the time of the East Asian crisis, saw a marked depreciation and highly increased flexibility of the INR. The third period exposes much tighter pegging again with low volatility and some small (but significant) weight on DUR. In the fourth period after March 2004, India moved away from the tight USD peg to a basket peg involving several currencies with greater flexibility (but smaller than in the second period). In this period, reserves in excess of 20% of GDP were held by the Reserve Bank of India (RBI), and a modest pace of reserves accumulation has continued.

The confidence intervals for the three break dates at 90% level are [1994-11-11, 1995-03-10], [1998-08-14, 1998-12-18], and [2003-11-28, 2004-04-02], respectively. These are again rather non-symmetric with tight bounds corresponding to

the low-variance regimes. (Recall that weekly data is employed and thus a difference of seven days is the tightest bound possible.)

The existing literature classifies the INR as a de facto pegged exchange rate to the USD in the period after April 1993 (Reinhart and Rogoff, 2004). Table 2 shows the fine structure of this pegged exchange rate; it supplies dates demarcating the four phases of the exchange rate regime; and it finds that by the fourth period, there was a basket peg in operation. This constitutes a statistically well-founded alternative to the existing classification schemes of the Indian exchange rate regime.

5. Summary

A formal inferential framework for data-driven assessment of the evolution of exchange rate regimes is presented. Based on a standard exchange rate regression model, statistical procedures for testing the stability of exchange rate regimes in historical data, monitoring exchange rate regimes in incoming data and dating breaks between exchange rate regimes are suggested. To simultaneously assess the stability of regression coefficients and error variance, an (approximately) normal regression model is adopted and embedded into a general unified structural change toolbox for (quasi-)likelihood-based regression models. All methods are based on the (negative) log-likelihood as the objective function or the corresponding estimating functions (likelihood scores), respectively. In particular, methods for estimating the breakpoints in such a model are extended from the corresponding techniques for least-squares regression. The tools are applied to investigate the changes in the regimes of two currencies: CNY and INR. For CNY, a 4-segment model is found for the time after July 2005 when China gave up on a fixed exchange rate to the USD. While CNY is closely linked to USD in all segments, there was a long period with somewhat increased flexibility and slight appreciation during 2006–2008 before the regime returned to a tight USD peg in 2009. For INR, a 4-segment model is found with a close linkage of INR to USD in the first three periods (with tight/flexible/tight pegging, respectively) before moving to a more flexible basket peg in spring 2004.

Acknowledgements

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