Does Precommitment Raise Growth? The Dynamics of Growth and Fiscal Policy*

May 14, 2003

Thomas Krichel	Paul Levine
Department of Economics	Department of Economics
University of Surrey	University of Surrey
Guildford GU2 5XH	Guildford GU2 7XH
United Kingdom	United Kingdom
T.Krichel@surrey.ac.uk	P.Levine@surrey.ac.uk

Abstract

We develop an endogenous growth model driven by externalities from both private and public capital. The government levies distortionary taxation to finance a publicly provided consumption good and public infrastructure. Firms face adjustment costs. We compare the optimal and time-consistent policies in a linear-quadratic approximation of the model. Although the time-consistent equilibrium is sub-optimal in terms of ex ante intertemporal welfare, it yields higher long-run growth and welfare, through an accumulation of assets by the state and a cut in government consumption.

Keywords: Endogenous growth; Dynamics; Fiscal policy; Commitment

JEL classification: E62; H53

*The authors are grateful to two anonymous referees of this journal for helpful comments on an earlier draft.

I Introduction

This paper studies a model of fiscal policy with endogenous growth which is dynamic in three ways. First, it models both private and public capital as stocks rather than flows. Second, it is non-Ricardian with distortionary taxes, finite lives and population growth. The choice between debt or taxation financing of a given path of government spending affects GDP growth and we do not impose a balanced budget. These two features imply that the model has transitory dynamics, a characteristic that is absent from many papers in the fiscal policy and endogenous growth literature. Third, the model allows for dynamic fiscal policy; in particular we allow governments to choose a different tax rate and spending level in each period.

The model incorporates a private sector and a government. The government may spend on a publicly-provided consumption good and on augmenting an infrastructure stock that provides for an externality in the production of firms. In the steady state, the government maintains this input in production as a constant fraction of GDP. Therefore the marginal productivity of capital is bounded away from zero and perpetual growth is possible. This type of model is pioneered by **?**). He models a single-good world where production is a function of labour in inelastic supply, a capital stock that does not depreciate, and the flow of public services. The main result of that paper is that welfare is maximized when growth maximized. Some of the subsequent literature has been concerned with welfare maximization in the context of more elaborate models where this result may no longer hold.

?) consider the case where both private and public capital stock depreciate fully during the period and preferences are logarithmic. Rather then assuming constancy of the tax rate, they derive the result that the optimal rate is constant. **?**) extends **?**) to include government consumption. It turns out that the government consumption is lower, and that government investment is higher, under welfare maximization than under growth maximization. This point is also emphasised by **?**) who introduce infrastructure as a stock rather than a flow. This is more realistic but makes an intertemporal welfare analysis analytically intractable. Despite these technical problems, **?**) show that an equilibrium exists under very general conditions, provided that the government can precommit to a sequence of government expenditures. An example for such an equilibrium is developed in ?). He models public capital as a stock, the services of which are subject to congestion. There are no adjustment costs, public and private capital can be costlessly transformed one into the other. In that way the model ends up with a single state variable which is the ratio of the two stocks. There is a fairly elaborate array of financing options, including consumption taxes, income taxes and lump-sum taxes or transfers. He first computes the first-best where a benevolent planner will directly allocate consumption and investment in the two stocks. The instruments of fiscal policy are sufficient to replicate the first best. During the transition to the optimum steady state, the private capital stock growth rate overshoots and the growth rate of the infrastructure stock undershoots. Thus both reach the common long-run growth rates from opposite sides.

There is a whole strand of the literature that does not attempt a full welfare analysis and therefore can reach analytical results for give policy changes. ?), ?), ?), and ?) all use a model with government capital to examine the effects of a given fiscal policy. For example, ?) model public capital as a stock to consider the case where it is subject to congestion. In the absence of congestion, an increase in the infrastructure stock, financed through a lump-sum tax, increases the long-run private capital stock if both factors are complements in production. If the degree of congestion is large, the increase in public infrastructure will lead to a fall in the private capital stock, provided that the substitutability of both factors is low. In a similar vein, ?) examine a variant of the basic model with endogenous labour supply and show that this model can account for much of the recent US growth experience. Finally there is a separate strand of literature that examines fiscal policy in real business cycle models, see for example ?).

Our model departs differs in significant ways from all the papers mentioned so far. First we use the non-Ricardian ?)-?)-?) demand framework for a realistic assessment of fiscal policy. Second, we allow for time-varying taxation and government spending *and* explicit policy optimisation by the government in a situation where the first-best can not be a achieved in equilibrium. Finally we characterize the optimum and time-consistent policy trajectories, even though the exact values are dependent on the parameters of the model. There is no free lunch of course—all these extensions require numerical simulation and therefore our results to not have the general power of analytical results.

The rest of the paper is organized as follows. Section II sets out our model. Section

III uses simulations on a calibrated model to compare the optimal precommitment fiscal policy with the time-consistent policy. Section IV concludes the paper.

II The Model

Our model treats both households and firms as intertemporal maximizers in a fairly standard fashion. We describe the behaviour of households, firms and the government in a closed economy.

Households

We consider a population of identical households who face a constant probability of death м per period. It grows at the exogenous rate г. All individuals enjoy logarithmic felicity $V(\mathfrak{t}) = \ln C(\mathfrak{t}) + \eta \ln G^c(\mathfrak{t})$ from consuming a private consumption commodity $C(\mathfrak{t})$ and a publicly provided consumption commodity $G^c(\mathfrak{t})$. They discount at a rate д and face a constant probability м of dying in each period. By virtue of this exponential lifetime assumption, their expected lifetime utility is independent of age and given by

$$U(\mathfrak{t}) = \sum_{\mathfrak{t}'=\mathfrak{t}}^{\infty} \left(\frac{1 - \text{м}}{1 + \text{д}} \right)^{\mathfrak{t}'-\mathfrak{t}} V(\mathfrak{t}') \tag{1}$$

where we use the uppercase letters to express magnitudes on a per-person level.

Every household is endowed with a unit of labour that she supplies inelasticly to the market in exchange for a post-tax wage $W(\mathfrak{t})$. At the end of any period $\mathfrak{t} - 1$, we can define her *human* wealth $H(\mathfrak{t} - 1)$ as the present value of the current and all future expected wages, discounted at the post-tax interest rate $r^{\tau}(\mathfrak{t}) = r(\mathfrak{t})\,(1 - \tau(\mathfrak{t}))$, where $\tau(\mathfrak{t})$ is the tax rate on all income (labour and capital) of households:

$$H(\mathfrak{t} - 1) = \sum_{\mathfrak{t}'=\mathfrak{t}}^{\infty} \frac{(1 - \text{м})^{\mathfrak{t}'-\mathfrak{t}+1}\, W(\mathfrak{t}')}{1 + r^{\tau}(\mathfrak{t}' - 1)} \tag{2}$$

Human wealth is the same for all living individuals, irrespective of their age, because they all face the same death rate and because the wage is not dependent on age. That does not mean, however, that households of all ages will have the same consumption, because recently born households have no non-human wealth, which they only start accumulating after birth. Non-human wealth $X(\mathfrak{t})$ takes the form of physical capital or government bonds, and because of arbitrage between both types of assets, they must earn the same return. At time $\mathfrak{t}$ the household born in $\mathfrak{t}' < \mathfrak{t}$ has some non-human wealth $X(\mathfrak{t}', \mathfrak{t}-1)$ at her disposal that she accumulated up to the end of period $\mathfrak{t}-1$. When the household dies she leaves an *unintentional* bequest, her non-human wealth, at the beginning of the period where death occurs. To model this set-up the following construction is introduced. There is an insurance company that takes the financial post-tax wealth of each dead household. It then distributes these assets as a premium п paid on the holdings of assets. If the insurance company has no operating cost, the premium satisfies the zero-profit condition: $1 + \text{п} - 1/(1 - \text{м}) = 0$. Hence we have the dynamics of non-human wealth as:

$$X(\mathfrak{t}', \mathfrak{t}) = \frac{(1 + r^{\tau}(\mathfrak{t}-1))\, X(\mathfrak{t}', \mathfrak{t}-1)}{1 - \text{м}} + W(\mathfrak{t}) - C(\mathfrak{t}) \tag{3}$$

If we solve the period-to-period budget constraint (3) forwards in time and make the conventional transversality assumption that the present value of future wealth will tend to zero, the lifetime budget constraint of a household born at time $\mathfrak{t}'$ is:

$$X(\mathfrak{t}, \mathfrak{t}) = \sum_{\mathfrak{t}'=\mathfrak{t}+1}^{\infty} \frac{(1 - \text{м})^{\mathfrak{t}'-\mathfrak{t}} \left[C(\mathfrak{t}') - W(\mathfrak{t}')\right]}{1 + r^{\tau}(\mathfrak{t}-1, \mathfrak{t}'-1)} = 0 \tag{4}$$

since there are to bequests and where we have defined the interest rate between period $\mathfrak{t}$ and $\mathfrak{t}'-1$ as:

$$1 + r^{\tau}(\mathfrak{t}, \mathfrak{t}'-1) = \prod_{\mathfrak{t}''=\mathfrak{t}}^{\mathfrak{t}'} (1 + r^{\tau}(\mathfrak{t}''-1)) \tag{5}$$

and $r^{\tau}(\mathfrak{t}, \mathfrak{t}) = r^{\tau}(\mathfrak{t})$. The household's problem is to maximize (1) under (4). The familiar first order condition is:

$$\frac{C(\mathfrak{t}')}{C(\mathfrak{t})} = \frac{1 + r^{\tau}(\mathfrak{t}-1)}{1 + \text{д}} \tag{6}$$

This completes the study of the individual household. All households of the same age are identical, but households of different ages have different non-human wealth. We therefore need to aggregate over different age levels. This leads to a "Yaari-Blanchard" demand function[1], which may be written as:

$$\begin{aligned} 0 = &\left(\frac{\text{м} + \text{г}}{1 + \text{г}} - \frac{1 + \text{д}}{1 - \text{м}}\right) c(\mathfrak{t}) + \frac{1 + r(\mathfrak{t}-1)\,(1 - \tau(\mathfrak{t}))}{1 + n(\mathfrak{t})}\, c(\mathfrak{t}-1) \\ &- \frac{(\text{м} + \text{д})\,(\text{м} + \text{г})}{(1 - \text{м})(1 + \text{г})}\, x(\mathfrak{t}) \end{aligned} \tag{7}$$

[1]Details of the aggregation procedure are given in the working paper version ?). See also ?) for a continuous-time version of (7). Note that in the Ricardian case $\text{м} = \text{г} = 0$ and (7) gives us the familiar Keynes-Ramsey rule.

Here the lower-case variable $c(\mathfrak{t})$ refers to aggregate level consumption in per-GDP form[2] $n(\mathfrak{t}) = [Y(\mathfrak{t}) - Y(\mathfrak{t}-1)]/Y(\mathfrak{t}-1)$ is the rate of growth of GDP.

Wealth is composed of physical capital $k(\mathfrak{t})$ and government debt $d(\mathfrak{t})$:

$$x(\mathfrak{t}) = d(\mathfrak{t}) + k(\mathfrak{t}) \tag{8}$$

The next two subsections deal with the accumulation of these assets.

Firms

Capital evolves under the impact of depreciation and investment as in (16). Investment is given by the solution of a profit maximization problem as follows. Imagine a large number of identical firms. We use uppercase notation for per firm levels. For every date $\mathfrak{t}' > \mathfrak{t}$, the problem of each firm is to choose investments $I(\mathfrak{t}')$, and employment $L(\mathfrak{t}')$ that maximize the discounted sum of future profits:

$$\sum_{\mathfrak{t}'=\mathfrak{t}}^{\infty} \frac{Y(\mathfrak{t}') - w(\mathfrak{t}')\,\epsilon(\mathfrak{t}')\,L(\mathfrak{t}') - I(\mathfrak{t}')\,[1 + a((I(\mathfrak{t}') - (n(\mathfrak{t}') + \delta)\,K(\mathfrak{t}'-1))/K(\mathfrak{t}'-1))]}{1 + r(\mathfrak{t}-1, \mathfrak{t}'-1)} \tag{9}$$

where $r(\mathfrak{t}-1, \mathfrak{t}'-1)$ is defined in the same way as (5).

The function $a(\cdot)$ in the expression (9) gives adjustment costs that the firm pays when investing. We make the usual assumptions that $a'(\cdot) > 0$, $a''(\cdot) > 0$, and $a(0) = 0$. Since $I(\mathfrak{t}) = (n + \delta)\,K(\mathfrak{t}-1)$ on a balanced growth path, the last assumption implies that there are no adjustment costs in this set-up.

Output is given by the Cobb-Douglas production function:

$$Y(\mathfrak{t}) = K(\mathfrak{t}-1)^{\alpha}\,[\epsilon(\mathfrak{t})\,L(\mathfrak{t})]^{1-\alpha} \tag{10}$$

Here $\epsilon(\mathfrak{t})$ is the efficiency of the labour force $L(\mathfrak{t})$. We adopt the approach to endogenous growth pioneered by **?**) and **?**) and allow the productivity of each worker to depend not only on the capital internal to the firm but also on externalities from the average capital available to the other firms $K(\mathfrak{t}-1)$, and from the infrastructure put in place by the government $K^{\mathrm{g}}(\mathfrak{t}-1)$, i.e.

$$\epsilon(\mathfrak{t}) = \bar{\epsilon}^{1/(1-\alpha)}\,\frac{K^{\mathrm{g}}(\mathfrak{t}-1)^{\gamma_1}\,K(\mathfrak{t}-1)^{1-\gamma_1}}{L(\mathfrak{t})}. \tag{11}$$

[2]All lower-case variables—apart from the interest, tax and growth rates—are in per GDP form. All stocks refer to end-of-period.

This feature of the model drives long-run endogenous growth. The aggregate production function now becomes:

$$Y(\mathsf{t}) = \bar{\epsilon}(\mathsf{t})\, K^{\mathrm{g}}(\mathsf{t}-1)^{1-\gamma_2}\, K^{\mathrm{g}}(\mathsf{t}-1)^{\gamma_2} \tag{12}$$

where $\gamma_2 = \alpha + (1-\alpha)\,(1-\gamma_1)$. In per-GDP form, this is the equation is written as

$$n(\mathsf{t}+1) = \bar{\epsilon}\, k^{\mathrm{g}}(\mathsf{t})^{1-\gamma_2}\, k(\mathsf{t})^{\gamma_2} - 1. \tag{13}$$

Performing the profit maximization, and aggregating over all firms, we get:

$$\begin{aligned} 0 = {} & \frac{\alpha\,(1+n(\mathsf{t}+1))}{k(\mathsf{t})} + \psi\,\frac{(1+n(\mathsf{t}+1))^3\, i(\mathsf{t}+1)^3}{k(\mathsf{t})^3} \\ & - \psi\,\frac{(n(\mathsf{t}+1)+\delta)\,(1+n(\mathsf{t}+1))^2\, i(\mathsf{t}+1)^2}{k(\mathsf{t})^2} \\ & + (1-\delta)\,\mathsf{q}(\mathsf{t}+1) - (1+r(\mathsf{t}))\,\mathsf{q}(\mathsf{t}) \end{aligned} \tag{14}$$

where q is Tobin's q. It is defined as

$$\begin{aligned} \mathsf{q}(\mathsf{t}) = {} & 1 + \frac{\psi}{2}\left(\frac{(1+n(\mathsf{t}))\, i(\mathsf{t}) - (\delta+n(\mathsf{t}))\, k(\mathsf{t}-1)}{k(\mathsf{t}-1)}\right)^2 \\ & + \frac{(1+n(\mathsf{t}))\, i(\mathsf{t})}{k(\mathsf{t}-1)}\,\psi\,\frac{(1+n(\mathsf{t}))\, i(\mathsf{t}) - (\delta+n(\mathsf{t}))\, k(\mathsf{t}-1)}{k(\mathsf{t}-1)}. \end{aligned} \tag{15}$$

Note that this implies that investment, like consumption, is forward-looking. The evolution of the capital stock is given by:

$$k(\mathsf{t}) = \frac{1-\delta}{1+n(\mathsf{t})}\, k(\mathsf{t}-1) + i(\mathsf{t}) \tag{16}$$

Government

Government debt—the second component of wealth in (8)—is issued by the government to satisfy its budget identity:

$$d(\mathsf{t}) = \frac{1+r(\mathsf{t}-1)}{1+n(\mathsf{t})}\, d(\mathsf{t}-1) + g(\mathsf{t}) - t(\mathsf{t}) \tag{17}$$

Tax revenue $t(\mathsf{t})$ is defined as

$$t(\mathsf{t}) = \tau(\mathsf{t})\left[1 - \frac{\delta\, k(\mathsf{t}-1)}{1+n(\mathsf{t})}\right] \tag{18}$$

where $\tau(\mathsf{t})$ is the tax rate. The term in square brackets assures that capital stock depreciation is tax-deductible. Government spending $g(\mathsf{t})$ is split into consumption spending $g^{\mathrm{c}}(\mathsf{t})$ and government investment $g^{\mathrm{i}}(\mathsf{t})$. Government investment generates the same kind of adjustment costs as private investment. Total government spending is therefore given by:

$$g(\mathfrak{t}) = g^{\mathrm{c}}(\mathfrak{t}) + g^{\mathrm{i}}(\mathfrak{t}) \left[1 + \frac{\psi}{2} \left(\frac{(1+n(\mathfrak{t}))\, g^{\mathrm{i}}(\mathfrak{t}) - (\delta + n(\mathfrak{t}))\, k^{\mathrm{g}}(\mathfrak{t}-1)}{k^{\mathrm{g}}(\mathfrak{t}-1)} \right)^2 \right] \tag{19}$$

Infrastructure is assumed to depreciate at the same rate as private capital. Therefore the evolution of infrastructure is given by:

$$k^{\mathrm{g}}(\mathfrak{t}) = \frac{1-\delta}{1+n(\mathfrak{t})}\, k^{\mathrm{g}}(\mathfrak{t}-1) + g^{\mathrm{i}}(\mathfrak{t}) \tag{20}$$

We assume that the government is perfectly benevolent; however there is no representative household in our overlapping generations model, but rather a spectrum of young and old households and those yet to be born. Following a suggestion by ?)[3] we use aggregate consumption to represent households of different generations, to arrive at a social welfare function $\tilde{u}$ for the government with the form:

$$\begin{aligned} \tilde{u}(\mathfrak{t}) &= \sum_{\mathfrak{t}'=\mathfrak{t}}^{\infty} \left(\frac{1-\text{м}}{1+\text{д}} \right)^{\mathfrak{t}'} \tilde{v}(\mathfrak{t}') \quad \text{where} \\ \tilde{v}(\mathfrak{t}') &= \ln c(\mathfrak{t}') + \eta \ln g^{\mathrm{c}}(\mathfrak{t}') + (1+\eta) \sum_{\mathfrak{t}''=\mathfrak{t}+1}^{\mathfrak{t}'} \ln(1+n(\mathfrak{t}'')) \end{aligned} \tag{21}$$

To derive the solvency constraint—as opposed to the identity (17)—for the government, first consider the "growth-adjusted" real interest rate over $[\mathfrak{t}-1, \mathfrak{t}]$ as $\rho(\mathfrak{t}) = (1 + r(\mathfrak{t}-1))/(1+n(\mathfrak{t})) - 1$. Then solving (17) forward in time, we transform the budget identity into a solvency constraint at time $\mathfrak{t}$, analogous to (4)

$$d(\mathfrak{t}-1) = \sum_{\mathfrak{t}'=0}^{\infty} \frac{t(\mathfrak{t}+\mathfrak{t}') - g(\mathfrak{t}+\mathfrak{t}')}{(1+\rho(\mathfrak{t}))\,(1+\rho(\mathfrak{t}+1)) \ldots (1+\rho(\mathfrak{t}+\mathfrak{t}'))} \tag{22}$$

provided that the transversality or "no-Ponzi" condition

$$\lim_{\mathfrak{t}' \to \infty} \frac{d(\mathfrak{t}+\mathfrak{t}')}{(1+\rho(\mathfrak{t}))\,(1+\rho(\mathfrak{t}+1)) \ldots (1+\rho(\mathfrak{t}+\mathfrak{t}'))} = 0 \tag{23}$$

holds. In (22) and (23) we assume that eventually $\rho(\mathfrak{t}) > 0$. This is a feature of the Yaari-Blanchard consumption/savings model and rules out dynamic inefficiency. According to (22) a government in debt with $d(\mathfrak{t}) > 0$ must, sometime in the future,

[3] They showed that a general optimization problem that takes account of generational diversity could be broken down into a problem of maximizing a function of aggregate consumption and a second problem of distributing aggregate consumption between generations.

run primary surpluses to be solvent. The transversality condition (23) does not require a *stable* debt/GDP ratio but merely that, in the long run, it does not increase faster than the growth-adjusted real interest rate $\rho(\mathfrak{t})$. However in a world with even very small departures from perfectly functioning capital markets, the notion of unbounded government debt/GDP ratios does not appeal. A stronger concept of solvency is that debt/GDP ratios stabilize. We enforce this condition through a small penalty attached to debt in the government's loss function which reflects the costs of issuing debt or acquiring assets if d is negative. We also include the cost of collecting taxes, therefore replacing $\tilde{v}$ in the social welfare function (21) by $\tilde{\tilde{v}}$:

$$\tilde{\tilde{v}}(\mathfrak{t}) = \tilde{v}(\mathfrak{t}) - \eta_d \left(d(\mathfrak{t})\right)^2 - \eta_\tau \left(\tau(\mathfrak{t})\right)^2 - \eta_{\Delta\tau} \left(\Delta\tau(\mathfrak{t})\right)^2 \tag{24}$$

The third term in (24) with a small value for η_d is sufficient to ensure a stable debt/GDP ratio, i.e. strong solvency. The final two terms penalize both large changes and large levels in the tax rate. We think of the inclusion of these extra terms as imposing a constraint on the liabilities or assets the government can acquire and on the extent of taxation it can impose in any one period. All these terms cover features not modelled explicitly.

The government's optimization problem at time $\mathfrak{t}$ is the maximization of (21), with $\tilde{v}$ replaced by $\tilde{\tilde{v}}$ given by (24). Maximisation takes place with respect to the government's choice variables $g^{\mathrm{c}}(\mathfrak{t}'), g^{\mathrm{i}}(\mathfrak{t}'), \tau(\mathfrak{t}')$, $\forall\ \mathfrak{t}' \geq \mathfrak{t}$, subject to the model of the private sector and the condition that commodity markets clear:

$$c(\mathfrak{t}) + i(\mathfrak{t}) \left[1 + \frac{\psi}{2} \left(\frac{i(\mathfrak{t})\,(1 + n(\mathfrak{t})) - (\delta + n(\mathfrak{t}))\,k(\mathfrak{t}-1)}{k(\mathfrak{t}-1)}\right)^2\right] + g(\mathfrak{t}) = 1 \tag{25}$$

This equation completes the model.

Equilibria

There are two equilibrium concepts depending on whether the government can precommit to a given trajectory for fiscal instruments over the future. If the government can precommit it can exercise the greatest leverage over the private sector. An announced path of instrument settings is credible and affects private sector behaviour immediately in the desired way. For instance the announcement of low taxes in the future will immediately raise savings, lower the real interest rate and increase private investment.

The solution to the optimal policy with precommitment is found by standard optimal control techniques using Lagrangian multipliers. Although the private sector is atomistic and therefore can not act strategically, the equilibrium concept corresponds to an open-loop Stackelberg equilibrium for dynamic games between strategic players described in chapter 7 of **?**).

When a government cannot commit itself to a future policy, it must act each period to maximize its welfare function, given that a similar optimization problem will be carried out in the next period. Formally, the policymaker maximizes at time $\mathfrak{t}$ a welfare function $\tilde{u}(\mathfrak{t})$ such that:

$$\tilde{u}(\mathfrak{t}) = \tilde{\tilde{v}}(\mathfrak{t}) + \varrho\, \tilde{u}(\mathfrak{t}+1) \tag{26}$$

where $\tilde{\tilde{v}}(\mathfrak{t})$ is the single-period felicity given in (24) and $\tilde{u}(\mathfrak{t})$ is evaluated on the assumption that an identical optimization exercise is carried out from time $\mathfrak{t}+1$ onwards. The solution to this problem is found by dynamic programming and, unlike the precommitment policy, leads to a time-consistent trajectory or rule for instruments[4]. This equilibrium concept corresponds to a feedback Nash equilibrium for dynamic games, see **?**), chapter 6. It has the property of being Markov-perfect—that is the government instruments and the private-sector forward-looking variables depend only on current values of the state variables. Following this solution procedure, a rational (utility-optimizing) government will never wish to deviate from the policies designed at the beginning of the planning period.

III Calibration and Results

Calibration

The model is calibrated around a steady-growth state fitted to the economy of the United States in 1990. The growth rate is the central variable of the model whose deduction as an endogenous variable would be subject to multiple numerical solutions. Therefore we first choose $n = 2.5\%$. We also fix $r = 5\%$. We chose the mortality $\textsc{m} = 2\%$, and overall population growth $\textsc{r} = 1\%$, to take account of immigration.

We collect basic national accounting data from the US Department of Commerce's Economic Bulletin Board. For the capital stock, we have data available from **?**) about

[4]See Appendix C of **?**) for details.

the net capital stock $K = 12706.7$ billion \$US. This is the figure we choose for the private capital stock, i.e. $k \approx 2.4$. Taking i to stand for fixed investment and using the observed figure for the capital stock k we calibrate the depreciation rate. We use our assumption that the rate of depreciation of private and public capital are equal to deduce the public infrastructure stock from the government expenditure on infrastructure. To estimate that expenditure, we collect data from the IMF Government Finance Statistics yearbook. We assume that the categories 4, 5 and 12 are the expenditure contributing to the capital stock of the government and aggregate federal state and local government expenditure. We can then compute r, the proportion of investment expenditure, as $\mathrm{r} \approx 36\%$. Adding the federal, state and local debt and dividing by GDP gives $d = 53.3\%$.

The exogenous parameters are summarized in the top two lines of Table 1. These lines also state two additional pieces of heuristics that we use to calibrate the model. First, we calibrate the government felicity parameter η on the ratio of public versus private consumption. This would be the result of a static optimization exercise: maximize $\ln C + \eta \ln G$ subject to $G + C$ being constant is $G = \eta C$; i.e., in per-GDP form $g = \eta\, c$. Notice that we are not assuming that dynamically optimal policies P or T are being pursued in the central calibration about which we linearise the model. The reason for this is that we need a linearised model in order to compute these regimes. However we do carry out sensitivity analysis on η and other parameters. Second we calibrate the relative productivity of the infrastructure to the ratio of infrastructure to the total (i.e. public plus private) capital stock.[5] The other lines in Table 1 illustrate how we derive the remaining values from the steady-state relationships.

Finally we choose the parameters η_τ, $\eta_{\Delta\tau}$, and η_d in (24). If we put $\eta_\tau = \eta_{\Delta\tau} = 0$ which implies no constraint on the size of the tax rate in any one period, but enforce strong solvency by setting η_d equal to a small value (in fact we find that 0.1 is sufficient for this purpose) we obtain optimal trajectories under precommitment for which the tax rate in the first period is over 100%, although the tax rate falls sharply thereafter. This

[5]There has been much debate about this parameter. The empirical results of ?) and ?) suggest that $1-\gamma_2$ is 39% and 34%, respectively. However we feel that these estimates should be upper bounds for $1-\gamma_2$ because other studies have found much lower value. In the extreme case, ?) suggest that $1-\gamma_2$ is not statistically significant from zero. We are however confident that our result carry over to a wide variety of scenarios because we have conducted extensive sensitivity analysis (see Appendix A).

$n = 2.5\%$	$r = 5\%$	$\text{м} = 2\%$	$\text{г} = 1\%$	$i = 15\%$	$\eta = g^{\mathrm{c}}/c$	$\approx 18\%$
$k = 241\%$	$g = 18\%$	$\text{г} = 36\%$	$d = 53.3\%$	$\bar{\epsilon} = 73\%$	$\gamma_2 = k/(k^{\mathrm{g}} + k)$	$\approx 75\%$

$$\delta = (1+n)\, i/k - n \qquad \approx 6\%$$

$$k^{\mathrm{g}} = \text{г}\, g\,(1+n)/(n+\delta) \qquad \approx 79\%$$

$$\bar{\epsilon} = (1+n)\, k^{\mathrm{g}-\gamma_2}\, k^{\gamma_2 - 1} \qquad \approx 73\%$$

$$\alpha = (r+\delta)\, k/(1+n) \qquad \approx 26\%$$

$$c = 1 - i - g \qquad \approx 67\%$$

$$g^{\mathrm{c}} = g\,(1 - \text{г}) \qquad \approx 12\%$$

$$\tau = d\,(r-n) + (1+n)\, g\,(1+n-\delta\, k) \qquad \approx 22\%$$

$$\text{д} = (1+n)\,(d + k(\text{м}\,(1+\text{г}) + c\big[r\,(1-\tau)\,(1+\text{г})\,(1-\text{м}) + (1-\text{м})$$
$$(\text{г} - \text{м}) + (\text{г}+\text{м})\,\text{м}\,(1-n)\big] / \left[c\,(1+\text{г}) + (\text{м}+\text{г})\,(k+d)\right] / (1+n) \qquad \approx 1.8\%$$

Table 1: Calibration

oddity reflects a number of deficiencies in our model including the absence of other tax distortions, the absence of explicit modelling of collection costs, political constraints on high tax rates etc, as well as the shortcomings of a linear-quadratic approximation. Fortunately quite small values of η_τ and $\eta_{\Delta\tau}$ remedy this feature of the simulation. We choose $\eta_\tau = \eta_{\Delta\tau} = 1$. These are small values because in our quadratic approximation the marginal rate of substitution between the consumption/GDP ratio c and τ along the modified utility curve is $-\eta_\tau\, \tau^*\, c^2/c^* = .12\,\eta_\tau$ for our calibration.

Results

Table 2 reports the long-run steady-state values of key variables for the precommitment (P) and time-consistent (T) regimes as deviations about the original steady state. In both regimes, debt becomes negative i.e., the government accumulates assets. Taxes fall in the long run, but they fall by more than government spending and the income that the assets accumulated generate make up for the difference. The most important difference between the regimes is the size of the long-run growth rate. Both regimes improve over the base line in terms of growth, but the growth rate in the T regime is over .5% higher. The immediate reason for this can be seen by examining changes in the per-GDP government and private capital stocks (k^g and k) respectively. In

	precommitment	time-cons.		precommitment	time-consistent
d_∞	−71.451	−75.492	τ_∞	−4.107	−9.269
n_∞	0.342	0.876	r_∞	0.005	0.301
c_∞	1.062	4.855	g^{c}_∞	−2.129	−7.589
i_∞	0.493	0.741	g^{i}_∞	0.574	1.993
k_∞	−2.942	−13.842	k^{g}_∞	4.009	16.576
$\tilde{u}(\infty)$	7.257	14.202	$\tilde{u}_0$	−217.6	−218.2

Table 2: Precommitment and Time-Consistent Policies. Variables are in per cent and measured as deviations about the original steady-state. For example, $n_{\mathrm{t}} = n(\mathrm{t}) - n$ where $n(\mathrm{t})$ is actual and n is steady-state growth. $\tilde{u}(\infty)$ is the steady-state welfare evaluated as in (28). $\tilde{u}_0$ is the quadratic approximation of welfare at time 0, as calculated by the simulation software. This number is not given as a per cent.

linear-deviation form (13) becomes

$$n_{t+1} = \frac{n}{k^g + k}\left[k^g_t + k_t\right] \tag{27}$$

using $\gamma_2 = k^{\mathrm{g}}/(k + k^{\mathrm{g}})$ from the calibration in Table 1. Hence from (27) growth increases if $k^g_t + k_t$ increases, i.e., if government capital stock increases by more than private capital stock decreases. This happens under both regimes, but more so under T, which is why growth also increases by more. However it should be noted that the low value for k—which is the private capital stock per GDP—is also a result of GDP expanding faster in the T regime and this is confirmed by the higher investment in this regime.

To complete the story we need to understand why capitals stocks change in this way. However first we consider the long-run implications of policy for welfare. The welfare of an individual who is born and lives in the steady state is equal to

$$\tilde{u}(\infty) = \frac{1 + \text{д}}{\text{м} + \text{д}}\left[\ln c + \eta \ln g^{\mathrm{c}} + \frac{(1+\eta)(1-\text{м})}{\text{д} + \text{м}} \ln(1+n)\right] \tag{28}$$

From (28) the steady-state intertemporal welfare depends on utility from current consumption, $c + \eta\, g^c$, and growth n. From Table 3.2 with $\eta = 0.18$ both these components of intertemporal welfare are higher in T relative to P in the long run for reasons we discuss below; hence long-run steady-state welfare is also higher.

Now we can evaluate the *growth-rate equivalent* of a steady-state regime change. Suppose that it would be possible to jump from the initial steady state to the steady state of the P and/or the T regime. By how much would the growth rate in the initial equilibrium have to raise in order to reflect that change? If we use the results from Table 2, we find that the long run of the optimal regime corresponds to an increase in growth by .35%. That is important, but not spectacular. The change from the initial steady state to the T regime steady state has a growth rate equivalent of .87%. The change from the steady-state of the P regime that of the T regime therefore corresponds to an increase in growth by .52%.

We have tested the properties that the long run of the T regime involves higher growth and welfare than the P regime over a wide range of parameter settings. Results are reported in Appendix A. Our sensitivity analysis suggests that our main findings are remarkably robust with respect to the calibration of the model.

Now let us return to the question of why T and P regimes differ in the accumulation of private and public capital. Figures 3.1 to 3.9 show the trajectories, for the key variables, all reported in deviation form about their baseline values. The overall profile of taxation and expenditure under the optimal (time-inconsistent) policy is as follows. A large burst of taxation in the first periods is followed by a decline in the tax rate. However we also see a later increase in taxation, such that the limiting steady-state tax rate is still positive. The explanation for this profile is quite familiar. The installed capital stock is predetermined at the beginning of the control period, i.e. the start of control is not expected by the private sector. Therefore a tax on that stock mimics a lump-sum tax. Using a heavy tax in the beginning therefore minimizes the welfare cost of taxation.[6] Thus for both P and T regimes government investment is financed by a combination of a increase in the tax rate, τ and a reduction in government consumption, g^{c}. Both these changes are concentrated at the beginning of the planning period. By implementing the tax increase in this way (in effect an initial tax surprise) its distortionary impact on private investment is contained. Private consumption (c) falls in the short run but increases in the long run. The fall in c and g^{c} crowd-in the increase in the public capital-GDP ratio k^{g} which more than compensates for the reduction in k,

[6] Note that this result is not dependent on the finite-life aspects of the model. All that matters is that taxation is distortionary.

brought about by an interest rate rise, so that growth increases.

All these changes take place in both P and T regimes when implemented starting at the baseline; however the time-consistency constraint means that the changes are more pronounced under T; c and g^c fall and taxation rises by more in the short-run under T and rise by more in the long-run, crowding-in a bigger increase in public capital stock and allowing for a much lower tax rate. Policy is of the general form of a sacrifice in the short run and reaping the benefit of higher growth and output level in the long run. Imposing time-consistency by optimizing first at the end of the planning period and working towards the beginning improves the welfare at the end in the T regime relative to P. Thus we end up with more initial sacrifice, but more long-run benefit in T.

Another way of viewing the time-consistency constraint is that it provides an incentive to continue to raise taxes until no more taxes are needed to finance expenditure. **?**) is an early contribution that established this result in a far simpler model without endogenous growth. Our study shows that the essence of **?**'s results carries over to a much more developed model incorporating endogenous growth. If we believe that the accumulation of debt is an important feature of observed economic policy, considering time-consistent policies does not bring the predictions of the model closer to the empirical facts; in fact it drives them away since asset accumulation of the government is larger.

The new element that we add to the picture is the decision between government consumption and investment expenditure. A naïve view would be to blame time consistency for insufficient investment. Our numerical experiments suggest that this is not correct and that in fact the time-consistent policy overaccumulates public capital. Loosely speaking we are adding a second layer of overinvestment into the dynamic behaviour. It is already known that for any given path of government expenditure, the time-consistent policy overaccumulates financial assets (with respect to the optimal policy). When we introduce the additional degree of freedom to allow the government either to consume or invest, we find overinvestment in physical assets.

If we believe that "out there in the real world" governments in fact underinvest, we can not take comfort from the time-consistent solution when searching for a theoretical underpinning for this view, unless we allow the government to discount *much* more heavily than the private sector. To fix ideas, let the government discount at a factor that is $\xi \leq 1$ times the discount factor of the private sector. Simulations show that as we decrease ξ, i.e., we make the government more and more impatient, the long-run asset accumulation in both regimes declines, but the asset position the time-consistent regime is more sensitive to the decline in ξ. Thus $\xi = .6$—a quite severe distortion—implies a long-run debt/GDP ratio of is -3% for the time-consistent regime, but -2% for the optimal regime. Note that the difference between growth rates is also affected. The optimal long run growth rate is .9% over the baseline, but the long-run time-consistent growth rate is .7% over the baseline. This suggests that the better long-run welfare improvement is valid when the government does not discount much more heavily than the private sector, but the stylized fact of government accumulation of debt can be captured by allowing the government to discount more heavily than the private sector.

IV Conclusion

Our main result is that precommitment can actually lead to *lower* long-run growth and welfare and the time-consistent solution is associated with an overaccumulation of assets by the government, unless the latter discounts more heavily than the private sector. Ex ante precommitment must yield higher intertemporal welfare and problems regarding implementation of optimal but time-inconsistent policies have focused on the establishment of some commitment mechanism that would make them credible. Our results suggest that the failure to find such a mechanism will actually be beneficial to future generations and can obviate the problems of short-termism associated with democratic decision-making.

We have provided intuition for our results in the context of a specific model. However if we step back from the discussion of the model, we can find some compelling reasons for why this result may be quite general. Imagine first an ex-ante optimal, time-inconsistent policy that involves "indulgence" initially, and "sacrifice" in the future. What would does the time-consistent policy look like? It is useful to consider a hypothetical "cheating" policy in which the time-inconsistent future policy trajectories are announced and believed by private sector, but the government then engages in re-optimization given these expectations. Then the tendency to indulge would continue during all earlier periods. But of course since past indulgence has eroded the possibility to indulge in the current period; as we move to the future, we indulge less in every period, because current indulgence reduces the possibility for future indulgence. In the long run we run down our opportunities to indulge to zero. Clearly the long-run welfare in such a cheating policy will be poor. In the time-consistent equilibrium the private sector anticipates the possibility of re-optimization so no cheating can occur. To achieve time-consistency and eliminate the incentive to cheat, the government must then offer more indulgence in the early period and more sacrifice later. Thus the reversal from indulgence to sacrifice will be more for the time-consistent policy than in the time-inconsistent case. The intertemporal welfare for the latter will be (by construction) better at the beginning of the planning period and it will also be superior to the time-consistent solution in the long-run.

But now assume that the opposite is true, that the optimal policy consists in making a sacrifice in the early periods and allow for indulgence in the later periods. Again, time-inconsistency in the form of an incentive to cheat and make more sacrifice in the early stage exists along this policy path. In the time-consistent solution there will be more sacrifice in the early periods, but in later periods, the sacrifice will bring fruit and allow for higher consumption possibilities. In contrast to be previous case, now the time-consistent policy brings *higher* welfare than the time-inconsistent policy in the long run.

Are most economic optimisation solution leading to trajectories of the "indulge then sacrifice" type or the "sacrifice then indulge" type? We not aware of any broad study of this question, but it seems to us that the latter type is much more prominent than the former. The latter situation arises for instance in models such as that is this paper, where there is capital accumulation problem and initial capital falls short of an overaccumulation level. It is also typically true in many models where the government can issue debt or accumulate assets and where initial government assets are smaller than the present value of government expenditure. Our conclusion should therefore hold in wide variety of models.

A Sensitivity analysis

In this appendix, we present a sensitivity analysis for the model. We change values for the fundamental parameters of the model and recompute the steady state that results from the optimal and time consistent policies. Note that we present the value that the variables take rather than the deviation from the steady state. This value will be different from the value in the original exercise because there is a different steady state and because there is a different policy that is associated with that steady state. There just reporting differences would have been misleading.

In the first column, we present the value of the fundamental variable that has changed. From the 3rd to the 13th column we give the value of the steady state of the variable indicated in the top row. For any shift in a fundamental parameter we report the steady state of the variable in the optimal regime 'P' and in the time-consistent regime 'T'.

All our results carry through these simulations.

		r	n	k	k^{g}	d	τ	c	i	g^{i}	g^{c}	u
$\gamma_2 =$	P	5.17	2.97	233	83.6	−17.9	18.8	67.7	15.4	7.23	9.66	11.16
.60240	T	5.84	3.81	212	95.9	−22	13.7	71.6	15.4	8.84	4.18	27.06
$\gamma_2 =$	P	4.89	2.76	241	80.2	−17.8	18.4	67.9	15.6	6.8	9.69	5.14
.90360	T	4.92	3.09	238	93.2	−21.7	13.1	71.8	16.0	8.10	4.12	5.47
$i =$	P	4.95	2.83	237	83.6	−20.0	18.1	71.3	12.4	7.13	9.22	11.05
.11984	T	5.07	3.04	232	90	−16.5	16.1	73.0	12.4	7.8	6.81	14.50
$i =$	P	5.05	2.87	238	82.6	−16.9	18.8	65.7	17.5	7.06	9.78	5.03
.16852	T	5.61	3.76	222	101	−27	11.1	71.1	18.1	9.27	1.58	7.99
$r =$	P	4.07	2.78	238	82.7	−16.5	19.3	67.7	15.4	7.01	9.98	22.23
.04000	T	4.41	3.38	227	95.7	−21.2	12.2	73	15.7	8.52	2.78	29.13
$r =$	P	5.93	2.91	238	83.3	−19.2	18.2	67.7	15.6	7.15	9.51	−3.56
.06000	T	6.18	3.38	227	95.2	−22.2	14.3	70.6	15.8	8.48	5.2	2.20
$\delta =$	P	5.04	2.82	239	93.1	−15.8	19	67.2	15.5	7.02	10.3	6.68
.0500	T	5.59	3.74	222	116	−28.0	10.7	73	16.3	9.41	1.33	6.88
$\delta =$	P	4.97	2.88	237	75.1	−20.4	18.2	68.6	15.5	7.16	8.81	8.04
.0700	T	5.15	3.18	230	82.3	−18.3	15.4	70.8	15.4	8.02	5.74	12.77

M =	P	5.02	2.86	238	83.4	−14.8	17.9	68.5	15.5	7.13	8.87	15.63
.01600	T	5.29	3.39	227	95.4	−24.3	12.6	72.3	15.8	8.5	3.49	21.72
M =	P	4.98	2.83	239	82.3	−21.9	19.3	67.1	15.5	7.01	10.3	0.22
.02400	T	5.30	3.38	227	95.5	−19.9	14.3	71	15.7	8.5	4.78	7.47
σ =	P	4.98	2.93	238	66.6	−18.2	16.6	69.3	15.7	5.74	9.32	11.52
.28616	T	5.24	3.54	227	76.8	−22.5	9.78	74.3	16.1	6.93	2.69	14.99
σ =	P	5.03	2.76	238	99.1	−17.5	20.4	66.5	15.3	8.49	9.7	3.01
.42924	T	5.35	3.23	227	114	−21.4	16.8	69.2	15.4	10.1	5.30	10.44
d =	P	5.02	2.81	238	82.9	−17.8	18.9	67.7	15.4	7.05	9.85	4.17
.42800	T	5.32	3.34	227	95.5	−21.5	13.9	71.5	15.7	8.47	4.41	11.16
d =	P	4.99	2.88	238	83	−17.9	18.4	67.9	15.6	7.10	9.47	8.08
.64200	T	5.28	3.41	227	95.5	−22.3	13.2	71.7	15.8	8.52	3.98	14.48
n =	P	5.02	2.3	238	87.4	−18.3	20.4	66.4	15.4	7.03	11.1	−15.76
.02000	T	5.38	2.98	225	103	−22.0	13.5	71.4	15.9	8.79	3.96	−8.18
n =	P	4.99	3.43	238	79.1	−17.0	16.3	69.7	15.6	7.16	7.49	36.08
.03000	T	5.26	3.82	229	88.9	−22.3	13.4	71.9	15.7	8.26	4.21	42.02
R =	P	5.01	2.85	238	83.2	−16.3	18.3	68.1	15.5	7.10	9.29	9.80
.00800	T	5.3	3.38	227	95.5	−22.9	13.1	71.9	15.8	8.49	3.87	16.16
R =	P	5	2.83	238	82.6	−19.6	19	67.5	15.5	7.04	10.0	2.69
.01200	T	5.30	3.38	227	95.5	−21.0	13.9	71.3	15.7	8.5	4.49	9.62
k =	P	5.06	2.82	209	82.3	−15.8	19.4	67.1	15.4	7.01	10.5	4.72
2.11	T	5.72	3.81	193	103	−27.6	11.4	72.8	16	9.46	1.73	10.24
k =	P	4.95	2.87	267	83.4	−19.4	17.9	68.6	15.6	7.14	8.68	7.53
2.71	T	5.12	3.19	259	91.7	−19.1	14.8	70.9	15.7	8.05	5.38	11.88
η =	P	5.03	2.79	238	82.9	−18.1	20.1	66.7	15.4	7.03	10.9	3.14
.196	T	5.34	3.30	227	95.5	−22.2	15.4	70.1	15.6	8.44	5.86	11.83
η =	P	4.97	2.91	238	83	−17.5	17	69.0	15.7	7.13	8.19	11.73
.156	T	5.25	3.47	227	95.5	−21.6	11.2	73.3	16	8.56	2.17	13.58

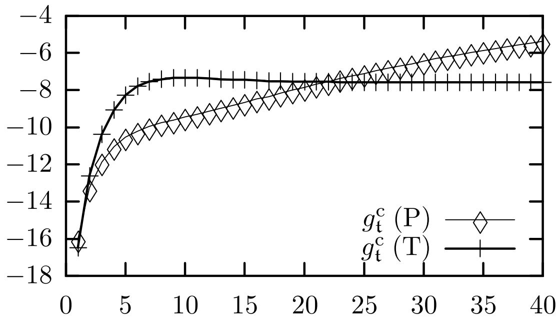

Figure 1: Government Consumption

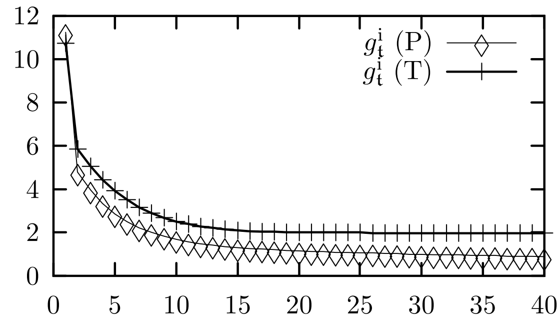

Figure 2: Government Investment

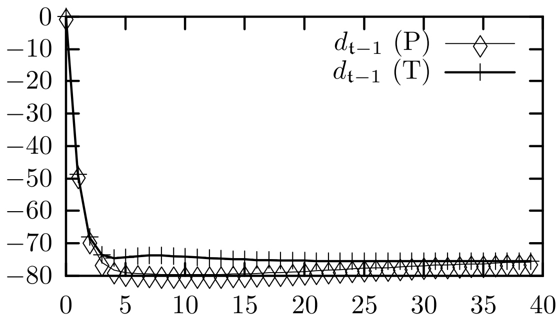

Figure 3: Debt

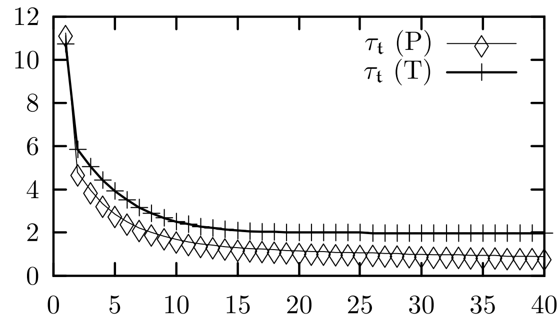

Figure 4: Tax rate

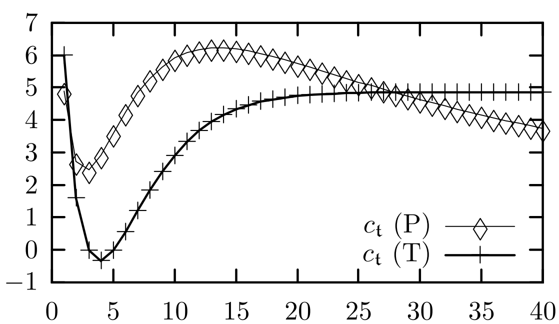

Figure 5: Consumption

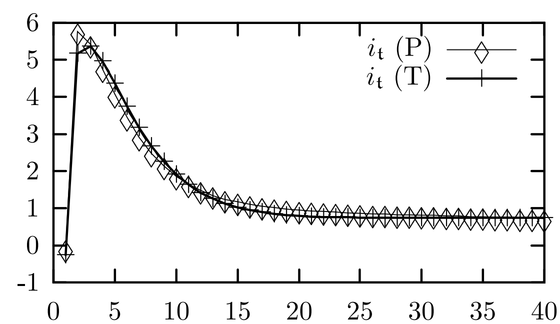

Figure 6: Investment

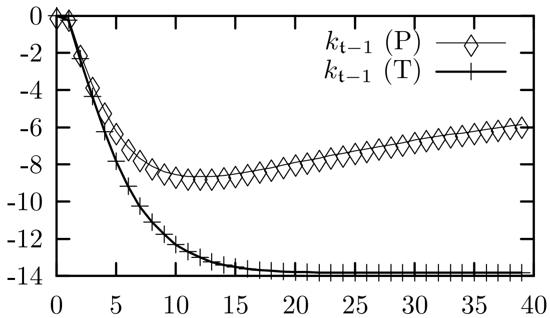

Figure 7: Capital

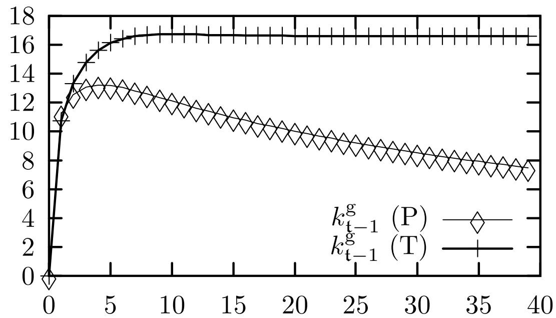

Figure 8: Infrastructure

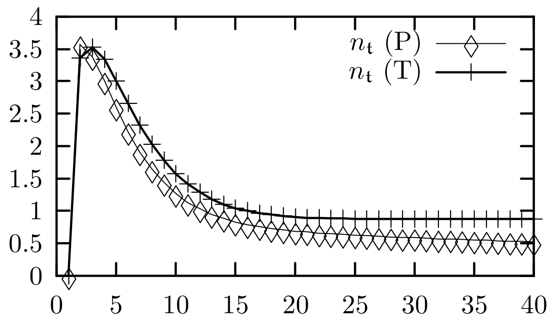

Figure 9: Growth rate

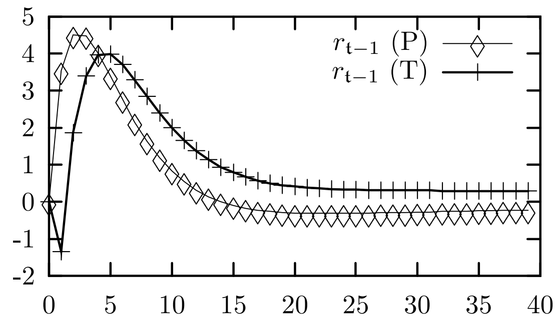

Figure 10: Interest rate