

# Testing, Monitoring, and Dating Structural Changes in Maximum Likelihood Models



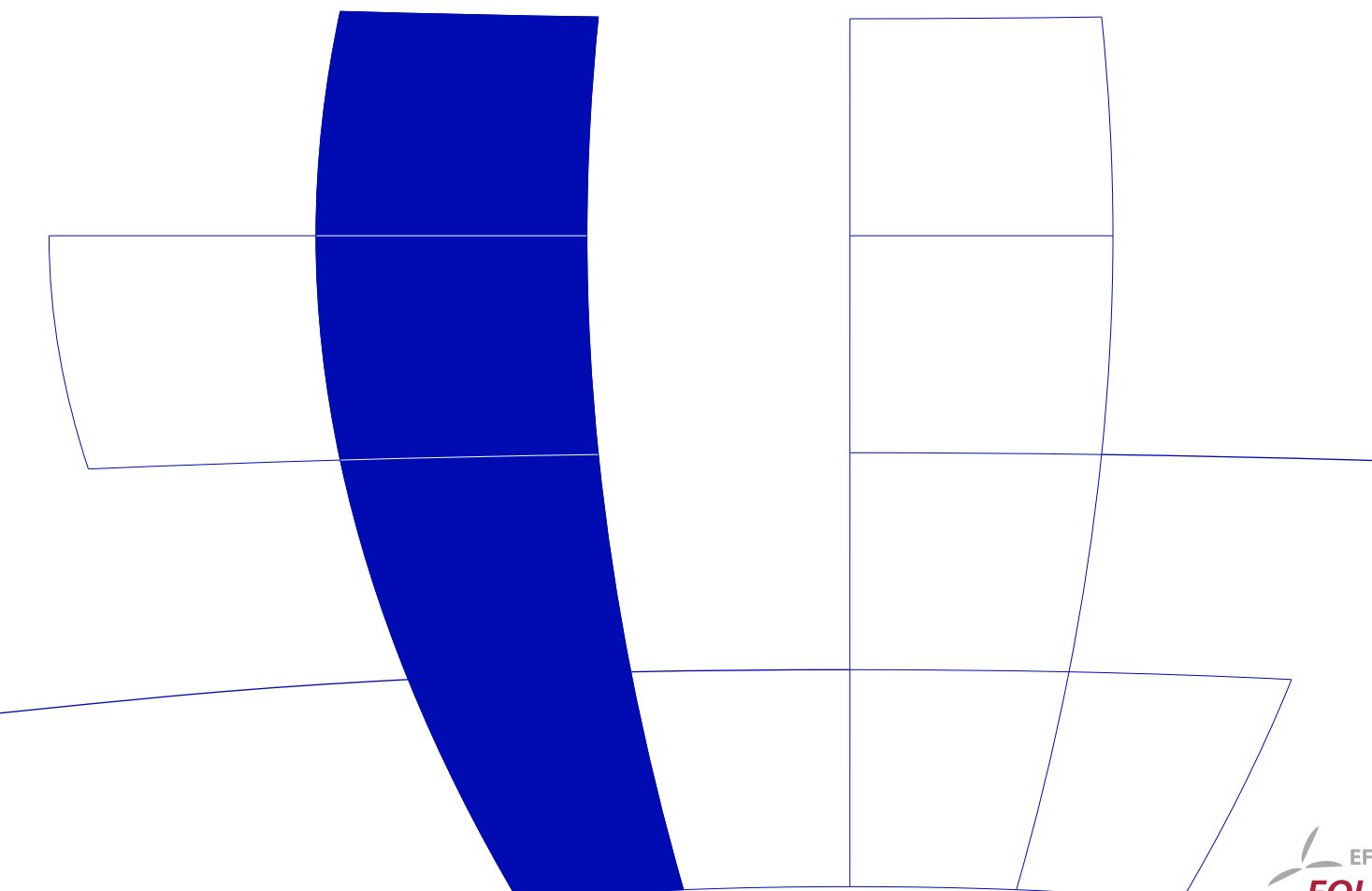
Achim Zeileis, Ajay Shah, Ila Patnaik

Department of Statistics and Mathematics  
Wirtschaftsuniversität Wien

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# Testing, Monitoring, and Dating Structural Changes in Maximum Likelihood Models

Achim Zeileis  
Wirtschaftsuniversität Wien

Ajay Shah  
NIPFP, New Delhi

Ila Patnaik  
NIPFP, New Delhi

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## Abstract

A unified toolbox for testing, monitoring, and dating structural changes is provided for likelihood-based regression models. In particular, least-squares methods for dating break-points are extended to maximum likelihood estimation. The usefulness of all techniques is illustrated by assessing the stability of *de facto* exchange rate regimes. The toolbox is used for investigating the Chinese exchange rate regime after China gave up on a fixed exchange rate to the US dollar in 2005 and tracking the evolution of the Indian exchange rate regime since 1993.

*Keywords:* parameter instability, structural change tests, monitoring, dating, foreign exchange rates, CNY, INR.

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## 1. Introduction

Techniques for assessing and modeling structural change have been of prime interest in theoretical and applied econometric research in the last two decades. Especially for least-squares regression a rich toolbox of methods has been established, encompassing tools for assessing model stability in the sample period used for estimation, or in future incoming observations, and for estimating breakpoints between sub-samples with different sets of parameters. These methods are also known as *testing* for structural change (see e.g., [Andrews 1993](#); [Hansen 2001](#); [Zeileis 2005](#)), *monitoring* structural change (see e.g., [Chu, Stinchcombe, and White 1996](#); [Zeileis, Leisch, Kleiber, and Hornik 2005](#)), and *dating* structural changes (see e.g., [Bai 1997](#); [Bai and Perron 2003](#)). While a unified suite of methods is well-established for least-squares regression, the corresponding techniques for likelihood-based regression models are somewhat scattered in the literature. Here, we provide a unifying view that illustrates how a functional central limit theorem for a likelihood-based model can be employed for testing, monitoring, and dating in this model. Specifically, this extends the dating algorithm for least-squares regression ([Bai and Perron 2003](#)) to maximum likelihood (ML) models.

All these techniques are applied to a problem which is of interest in the international economics literature: classification of exchange rate regimes. Analysis of exchange rate regimes received increasing interest during the last decade when it became clear that the *de jure* exchange rate regime in a country, as announced by the central bank, often differs from the *de facto* regime in operation (see e.g., [Reinhart and Rogoff 2004](#); [Levy-Yeyati and Sturzenegger 2003](#); [Bubula and Ötoker-Robe 2002](#)). The methods for identifying the exchange rate regime in the literature have often lacked inferential foundations. Here, we provide a formal approach to

analyzing this problem using the previously established unified structural change techniques. This involves a linear regression, but as the error variance is of crucial interest, it is incorporated as a full model parameter by adopting an (approximately) normal model estimated by (quasi-)maximum likelihood. We show applications for two currencies, the Chinese yuan CNY and the Indian rupee INR.

## 2. A unified structural change toolbox

In this section, we first outline the model frame for regression models estimated by ML along with the usual central limit theorem (CLT). For structural change methods, this CLT needs to be extended to a functional CLT (FCLT) based on which testing procedures (in historical samples), monitoring procedures (sequential tests for incoming data), and confidence intervals for breakpoints can be established. This yields a unified structural change toolbox for ML models and also provides some insights for applicability to models with other objective functions and estimating functions, respectively.

### 2.1. Model

We assume  $n$  observations of some dependent variable  $y_i$  and a regressor vector  $x_i$ , such that the  $y_i$  are following some distribution  $F$  with  $k$ -dimensional parameter  $\theta_i$ , conditional on the regressors  $x_i$ .

$$y_i | x_i \sim F(\theta_i) \quad (i = 1, \dots, n). \quad (1)$$

The ordering of the observations usually corresponds to time. Then, the hypothesis of interest is “parameter stability”, i.e.,

$$H_0 : \theta_i = \theta_0 \quad (i = 1, \dots, n) \quad (2)$$

that should be tested against the alternative that the parameter  $\theta_i$  changes over time.

If the parameters  $\theta$  in such a model are stable, they are typically estimated by minimizing a suitable objective function  $\Psi(y_i, x_i, \theta)$  or, equivalently, solving the corresponding first-order conditions based on the corresponding derivative  $\psi(y_i, x_i, \theta) = \partial\Psi(y_i, x_i, \theta)/\partial\theta$ :

$$\operatorname{argmin}_{\theta \in \Theta} \sum_{i=1}^n \Psi(y_i, x_i, \theta) = \hat{\theta}, \quad (3)$$

$$\sum_{i=1}^n \psi(y_i, x_i, \hat{\theta}) = 0. \quad (4)$$

To obtain the ML estimator, the objective function is the negative log-likelihood (NLL)  $\Psi_{\text{NLL}}(y_i, x_i, \theta) = -\log f(y_i | x_i, \theta)$  corresponding to the distribution  $F$ . We adopt this objective function throughout the paper but note that many of the methods discussed below can be applied straightforwardly to other M-type estimators such as ordinary least squares (OLS), nonlinear least squares (NLS), or quasi-ML. However, it is crucial that the estimating function  $\psi$  is correctly specified while misspecification of the full likelihood can often be remedied.

Under standard regularity conditions, e.g., as given in [White \(1994, Theorem 6.10, p. 104\)](#) or [Cameron and Trivedi \(2005, Chapter 5\)](#), a central limit theorem (CLT) holds:

$$\sqrt{n}(\hat{\theta} - \theta_0) \xrightarrow{d} \mathcal{N}(0, A_0^{-1} B_0 A_0^{-1}), \quad (5)$$

i.e.,  $\hat{\theta}$  is asymptotically normal with mean  $\theta_0$  and a sandwich-type covariance matrix with components

$$A_0 = \text{plim } n^{-1} \sum_{i=1}^n \mathbb{E}[-\psi'(y_i, x_i, \theta_0)], \quad (6)$$

$$B_0 = \text{plim } n^{-1} \sum_{i=1}^n \text{VAR}[\psi(y_i, x_i, \theta_0)], \quad (7)$$

where  $\psi'$  is the derivative of  $\psi$ , again with respect to  $\theta$ .

Under various sets of assumptions (see e.g., [Andrews 1993](#)), this CLT can be extended to a functional CLT (FCLT): The empirical fluctuation process  $efp(\cdot)$ , defined as the decorrelated partial sum process of the empirical estimating functions, converges to a  $k$ -dimensional Brownian bridge  $W^0(\cdot)$  on the interval  $[0, 1]$  (see e.g., [Zeileis 2005](#)).

$$efp(t) = \hat{B}^{-1/2} n^{-1/2} \sum_{i=1}^{\lfloor nt \rfloor} \psi(y_i, x_i, \hat{\theta}) \quad (0 \leq t \leq 1) \quad (8)$$

$$efp(\cdot) \xrightarrow{d} W^0(\cdot). \quad (9)$$

The Brownian bridge can also be written as  $W^0(t) = W(t) - tW(1)$ , where  $W(\cdot)$  is a standard  $k$ -dimensional Brownian motion. Furthermore,  $\hat{B}$  is some suitable estimator of  $B_0$ , e.g., the outer product of the empirical estimating functions or some HC (heteroskedsticity consistent) or HAC (heteroskedasticity and autocorrelation consistent) estimator to allow for certain deviations from the correct specification of the likelihood.

While the empirical fluctuation process  $efp(\cdot)$  is governed by the FCLT under the null hypothesis of parameter stability, its fluctuation should generally be increased under the alternative of structural change. In particular, the process typically exhibits peaks at the times changes in  $\theta_i$  occur. Hence,  $efp(\cdot)$  and the associated FCLT are the basis for the inference presented in the remainder of this section.

## 2.2. Testing

The classical question in structural change analysis is whether the true model parameter  $\theta$  is really stable throughout the sample period  $i = 1, \dots, n$ . Thus, testing for structural change is concerned with testing the null hypothesis  $H_0$  from Equation 2 against the alternative that  $\theta_i$  changes over time. Various patterns of change are conceivable, e.g., single or multiple breaks or random walks etc.

Deviations from parameter stability can be brought out by assessing deviations of the empirical estimating functions  $\psi(y_i, x_i, \hat{\theta})$  from their zero mean. This is done by applying some aggregation functional  $\lambda(\cdot)$  to the empirical fluctuation process  $\lambda(efp)$ , yielding a univariate test statistic. The corresponding limiting distribution is the same functional (or its asymptotic counterpart) applied to a Brownian bridge  $\lambda(W^0)$  so that critical values and  $p$  values can be derived. This approach encompasses a wide collection of structural change tests suggested in the literature, see [Zeileis \(2005\)](#) for a detailed discussion.

For brevity, we provide here just a few important special cases:

$$S_{dmax} = \sup_{t \in [0,1]} \|efp(t)\|_{\infty}, \quad (10)$$

$$S_{CvM} = n^{-1} \sum_{i=1}^n \|efp(i/n)\|_2^2, \quad (11)$$

$$S_{MOSUM} = \sup_{t \in [0,1-h]} \|efp(t+h) - efp(t)\|_{\infty}, \quad (12)$$

$$S_{\text{sup } LM} = \sup_{t \in [\pi, 1-\pi]} \frac{\|efp(t)\|_2^2}{t(1-t)}. \quad (13)$$

The double maximum statistic  $S_{dmax}$  (i.e., the maximum over parameters and time) is particularly useful for exploratory purposes because it can easily be visualized along with critical values (derived from the distribution of the maximum of a Brownian bridge) so that timing of a structural change and the parameter affected by it can be identified graphically. However, this test might have poor power in the presence of a random walk alternative or multiple breaks. In such a situation, a Cramér-von Mises statistic  $S_{CvM}$  as in the Nyblom-Hansen test (Nyblom 1989; Hansen 1992) or a MOSUM statistic  $S_{MOSUM}$  with bandwidth  $h$  would be more suitable. The statistic  $S_{\text{sup } LM}$  (Andrews 1993) is the supremum of LM statistics on the interval  $[\pi, 1 - \pi]$  (for some trimming parameter  $\pi$ ) and is particularly well-suited for single break alternatives.

### 2.3. Monitoring

Given that a stable model could be established for observations  $i = 1, \dots, n$ , it is natural to ask whether this model remains stable for future incoming observations  $i > n$ . More formally, monitoring (Chu *et al.* 1996) tests the null hypothesis

$$H_0 : \theta_i = \theta_0 \quad (i > n), \quad (14)$$

sequentially against changes in the so-called monitoring period  $i > n$  (or the scaled  $t > 1$ ).

Based on the tools from the previous section, an extension to the monitoring situation is fairly straightforward. The empirical fluctuation process  $efp(t)$  is simply continued in the monitoring period by computing the empirical estimating functions for each new observation (using the parameter estimates from the stable history period  $[0, 1]$ ) and updating the cumulative sum process. This is still governed by an FCLT on an extended interval  $[0, T]$  with  $T > 1$  (Zeileis 2005). Based on this FCLT, a testing procedure can be established that re-computes the functional  $\lambda(efp(t))$  for each new observation and rejects the null hypothesis from Equation 14 if it exceeds some critical value  $b(t)$  for any  $t > 1$ . As this is a sequential testing procedure, not only a single critical value is required but a function  $b(t)$  that can be interpreted as a boundary function for the empirical fluctuation process. To yield a level  $\alpha$  testing procedure, it needs to fulfill

$$1 - \alpha = \mathbb{P}(\lambda(W^0(t)) \leq b(t) \mid t \in [1, T])$$

Various combinations of functionals  $\lambda$  and boundaries  $b$  are conceivable (Chu *et al.* 1996; Horváth, Hušková, Kokoszka, and Steinebach 2004; Zeileis *et al.* 2005; Zeileis 2005) that can direct power against changes occurring early or late in the monitoring period  $t > 1$  or that try to spread the power evenly.

For the applications in Section 3 we adopt a maximum functional and a linear boundary function  $b(t) = \pm c \cdot t$  as suggested in Zeileis *et al.* (2005) that spreads the power rather evenly. More precisely, we detect a change and reject the null hypothesis if

$$\|efp(t)\|_\infty > c \cdot t \quad \text{for any } t \in [1, T]$$

where the critical value  $c$  can be obtained from (Zeileis *et al.* 2005, Table III) and applying a Bonferroni correction.

## 2.4. Dating

If there is evidence for parameter instability in the regression model, a natural question is to ask when and how the parameters changed. Often, a reasonable approximation is to adopt a segmented regression model, i.e., assume stable sets of parameters  $\theta^{(j)}$  exists for  $j = 1, \dots, m+1$  segments that are mutually exclusive and cover the sample period. More formally,  $\theta^{(j)}$  holds for the observations  $i = i_{j-1} + 1, \dots, i_j$  where  $\{i_1, \dots, i_m\}$  are the  $m$  breakpoints and, by convention,  $i_0 = 0$  und  $i_{m+1} = n$ .

The goal of dating is to determine estimates of the the  $m$  breakpoints and the  $m+1$  segment-specific parameters  $\theta^{(j)}$ , often followed by a subsequent model selection for the number of breakpoints  $m$ . If the breakpoints were known, estimation of the parameters  $\theta^{(j)}$  would be easy: they can be obtained by solving Equation 3 (or 4) in the  $j$ -th segment. The overall segmented objective function based on  $\Psi$  is then given by

$$\begin{aligned} PSI(i_1, \dots, i_m) &= \sum_{j=1}^{m+1} psi(i_{j-1} + 1, i_j), \\ psi(i_{j-1} + 1, i_j) &= \sum_{i=i_{j-1}+1}^{i_j} \Psi(y_i, x_i, \hat{\theta}^{(j)}), \end{aligned}$$

where  $psi(i_{j-1} + 1, i_j)$  is the minimal value of the objective function for the model fitted on the  $j$ -th segment with associated parameter estimate  $\hat{\theta}^{(j)}$ . Dating then tries to find the global optimizers  $\hat{i}_1, \dots, \hat{i}_m$  of the segmented objective function, i.e., solving

$$(\hat{i}_1, \dots, \hat{i}_m) = \underset{(i_1, \dots, i_m)}{\operatorname{argmin}} PSI(i_1, \dots, i_m) \quad (15)$$

subject to a minimal segment size constraint  $i_j - i_{j-1} + 1 \geq n_h \geq k$ . The minimal segment size is either chosen directly or derived from some trimming  $h$  as  $n_h = \lfloor nh \rfloor$ . The optimal (with respect to  $\Psi$ ) set of breakpoints from Equation 15 is called  $m$ -partition  $\mathcal{I}_{m,n} = \hat{i}_1, \dots, \hat{i}_m$ .

### Estimation

Direct optimization in Equation 15 by exhaustive search over all conceivable partitions is of order  $O(n^m)$  and hence computationally burdensome. Fortunately, the Bellman principle of optimality can be applied to the problem as the following recursion holds:

$$PSI(\mathcal{I}_{m,n}) = \min_{mn_h \leq i \leq n - n_h} [PSI(\mathcal{I}_{m-1,i}) + psi(i + 1, n)]. \quad (16)$$

Therefore, a dynamic programming approach can be employed that solves the global minimization in  $O(n^2)$ . Bai and Perron (2003) describe in detail such a dynamic programming

algorithm for minimizing the segmented residual sum of squares (RSS) in linear regression models. In fact, the same algorithm can be applied for computing the more general class of estimators considered here because the objective function is additive in the observations (Hawkins 2001). More precisely, for computing the ML estimators in some likelihood model (instead of OLS estimators in the linear regression model) their objective function  $\Psi_{\text{RSS}}$  just has to be replaced by the negative log-likelihood  $\Psi_{\text{NLL}}$ :

$$\begin{aligned}\Psi_{\text{RSS}}(\theta) &= (y_i - x_i^\top \theta)^2, \\ \Psi_{\text{NLL}}(\theta) &= -\log f(y_i | x_i, \theta).\end{aligned}$$

Essentially, the algorithm first computes a triangular matrix with  $\text{psi}(i, j)$  for all  $j - i \geq \lfloor nh \rfloor$  and  $i = 1, \dots, n - \lfloor nh \rfloor + 1$ . Based on this matrix, the problem from Equation 15 can be solved by exploiting Equation 16 for any number of breakpoints  $m$  (if  $(m + 1)\lfloor nh \rfloor < n$ ).

### *Model selection*

Using this algorithm, the optimal (with respect to  $\Psi$ ) segmentation can be computed if the number of breakpoints  $m$  is known. In practice, however,  $m$  typically needs to be chosen based on the observed data as well. One solution to this problem is to compute the optimal segmentations for a sequence of breakpoints  $m = 0, 1, \dots$  (which can all be computed from the same triangular matrix mentioned above) and to choose  $m$  by optimizing some information criterion  $IC(m)$ . If the segmentations are likelihood-based, such information criteria are easily available. Thus, if  $PSI(\mathcal{I}_{m,n})$  is based on  $\Psi_{\text{NLL}}$  we call it  $NLL(\mathcal{I}_{m,n})$  and computation of information criteria is straightforward:

$$IC(m) = 2 \cdot NLL(\mathcal{I}_{m,n}) + \text{penalty} \cdot ((m + 1)k + m),$$

with different penalties leading to different information criteria. Bai and Perron (2003) consider two different criteria, the BIC and a modified BIC as suggested by Liu, Wu, and Zidek (1997):

$$\begin{aligned}\text{penalty}_{\text{BIC}} &= \log(n), \\ \text{penalty}_{\text{LWZ}} &= \alpha \cdot \log(n)^{2+\delta}.\end{aligned}$$

In our empirical studies below, we follow the recommendations of Bai and Perron (2003) and Liu *et al.* (1997) and use the LWZ criterion in our empirical studies, setting the parameters  $\alpha = 0.299$  and  $\delta = 0.1$  so that the LWZ penalty is higher than in the BIC for  $n > 20$ .

### *Confidence intervals*

In addition to point estimation, confidence intervals for the true segment-specific parameters  $\theta_0^{(j)}$  and the true breakpoints  $i_j^0$  are of interest. Hence, some further extensions of the results of Bai and Perron (2003) are discussed here. As the breakpoint estimates  $\hat{i}_j$  converge at the faster rate  $n$ , the standard  $\sqrt{n}$  asymptotics from Equation 5 still hold for  $\hat{\theta}_{(j)}$  with segment-specific matrices  $A_0^{(j)}$  and  $B_0^{(j)}$  (analogous to Equations 6 and 7), respectively. Both can be estimated in the usual way, e.g., by computing a HC or HAC estimate from the observations in segment  $j$ . If the likelihood can be assumed to be correctly specified, then  $A_0^{(j)} = B_0^{(j)}$  corresponds to the Fisher information and is usually estimated by the Hessian matrix.

Confidence intervals for the true breakpoints  $i_j^0$  can also be derived if, following [Bai and Perron \(2003\)](#), an asymptotic framework is adopted where the magnitudes of the changes  $\Delta_j = \theta_0^{(j+1)} - \theta_0^{(j)}$  converges to zero as the sample size increases. Then the distribution of the breakpoint estimates is given by

$$\frac{\Delta_j^\top A_0^{(j)} \Delta_j}{\Delta_j^\top B_0^{(j)} \Delta_j} (\hat{i}_j - i_j^0) \xrightarrow{d} \operatorname{argmax}_t V^{(j)}(t) \quad (j = 1, \dots, m) \quad (17)$$

where  $V^{(j)}(\cdot)$  is a stochastic process defined by

$$V^{(j)}(t) = \begin{cases} W_1^{(j)}(-t) - |t|/2, & \text{if } t \leq 0 \\ \sqrt{\xi_j}(\phi_2^{(j)}/\phi_1^{(j)})W_2^{(j)}(t) - \xi_j|t|/2, & \text{if } t > 0 \end{cases}.$$

$W_1^{(j)}$  and  $W_2^{(j)}$  are independent Brownian motions and  $\xi_j = (\Delta_j^\top A_0^{(j+1)} \Delta_j)/(\Delta_j^\top A_0^{(j)} \Delta_j)$ ,  $\phi_1^{(j)} = (\Delta_j^\top B_0^{(j)} \Delta_j)/(\Delta_j^\top A_0^{(j)} \Delta_j)$ , and  $\phi_2^{(j)} = (\Delta_j^\top B_0^{(j+1)} \Delta_j)/(\Delta_j^\top A_0^{(j+1)} \Delta_j)$ . A closed form solution for the distribution function of  $\operatorname{argmax}_t V^{(j)}(t)$  is provided by [Bai \(1997, Appendix B\)](#). Hence, all that is required in addition for computing confidence intervals are estimates  $\hat{\Delta}_j = \hat{\theta}^{(j+1)} - \hat{\theta}^{(j)}$ , as well as  $\hat{A}^{(j)}$ , and  $\hat{B}^{(j)}$  which can be derived as above.

The arguments for deriving Equation 17 are completely analogous to [Bai and Perron \(2003, Section 4\)](#) and [Bai \(1997, Section II.C\)](#) if the role of regressors/disturbances in the linear regression model is replaced by the estimating functions in the likelihood model. Specifically, the estimating functions in an OLS regression are given by  $\psi(y_i, x_i, \theta) = x_i(y_i - x_i^\top \theta) = x_i u_i$  and the corresponding derivative is  $\psi'(y_i, x_i, \theta) = x_i x_i^\top$ . Consequently, the basic assumption is that an FCLT as in Equation 9 has to hold for each segment  $j$ , corresponding to [Bai \(1997, Assumption 9\)](#).

### 3. Application to exchange rate regime analysis

In the last decade, it has been revealed that the *de jure* exchange rate regime in a country, as announced by the central bank, often differs from the *de facto* regime in operation. This has motivated a small literature on data-driven methods for the classification of exchange rate regimes (see e.g., [Reinhart and Rogoff 2004](#); [Levy-Yeyati and Sturzenegger 2003](#); [Bubula and Ötoker-Robe 2002](#)). Broadly speaking, exchange rate regimes range from *floating*, i.e., the currency is allowed to fluctuate based on market forces, *pegged*, i.e., the currency has limited flexibility when compared with a basket of currencies or a single currency, or *fixed*, i.e., the currency has a fixed parity to another currency.

A valuable tool for understanding the *de facto* exchange rate regime in operation is a linear regression model based on cross-currency exchange rates (with respect to a suitable numeraire) popularized by [Frankel and Wei \(1994\)](#) and hence also called the Frankel-Wei model. To understand the *de facto* exchange rate regime in a given country in a given time period, researchers and practitioners can easily fit this regression model to a given data window, or use rolling data windows. However, such a strategy lacks a formal inferential framework for determining changes in the regimes. Hence, we provide such a framework by applying the structural change methods presented above to answer the following questions: 1. Testing: Is a given exchange rate model stable within the time period in which it was established?



2. Monitoring: If it is stable, does it remain stable for future incoming observations? 3. Dating: If it is not stable, when and how did the exchange rate regime change?

After briefly establishing the notation for this model, we illustrate the methods by investigating the exchange rate regimes of China and India.

### 3.1. Exchange rate regression

To determine whether a certain currency is pegged to (a basket of) other currencies, the exchange rate regression (Frankel and Wei 1994) employs a standard linear regression model

$$y_i = x_i^\top \beta + u_i \quad (i = 1, \dots, n), \quad (18)$$

in which the  $y_i$  are returns of the target currency and the  $x_i$  are vectors of returns for a basket of  $c$  currencies plus a constant.

When a country runs a fixed exchange rate, one element of  $\beta$  is 1 and the remaining elements are zero, and the error variance is  $\sigma^2 = 0$ . When a country runs a pegged exchange rate against one currency, one element of  $\beta$  is near 1, the remaining elements are near zero, and  $\sigma^2$  takes low values. With a basket peg,  $\sigma^2$  takes low values, and the coefficients  $\beta$  correspond to weights of the basket. With a floating rate,  $\sigma^2$  is high, and the  $\beta$  values reflect the natural current account and capital account linkages of the country.

For both  $y_i$  and  $x_i$ , we use log-difference returns (in percent) of different currencies as computed by  $100 \cdot (\log p_i - \log p_{i-1})$ , where  $p_i$  is the price of a currency at time  $i$  and given in CHF (Swiss franc) as the numeraire currency. Hence, in the following, we use ISO 4217 abbreviations to denote currency returns computed from prices in CHF. For the numeraire, other choices are conceivable (e.g., other currencies, special drawing rights, or gold); but for pegged exchange rate regimes the results are typically not very sensitive to the choice of the numeraire (Frankel and Wei 1994, 2007).

Assessing the stability of an exchange rate regression might seem trivial because it is a linear regression model typically estimated by OLS for which application of all structural change techniques is well-established practice. However, the error variance  $\sigma^2$  (capturing the flexibility of the exchange rate regime in operation) is of crucial interest here and has to be treated as a full model parameter and not just a nuisance parameter as in most linear regressions and associated structural change methods. Specifically, RSS-based structural change techniques (such as Bai and Perron 2003) are insensitive to changes in  $\sigma^2$ , if it is not included explicitly. Therefore, we adopt a normal model, i.e.,  $f(y|x, \beta, \sigma^2) = \phi((y - x^\top \beta)/\sigma)/\sigma$  where  $\phi(\cdot)$  is the standard normal density function. This has the full combined parameter  $\theta = (\beta^\top, \sigma^2)^\top$  of length  $k = c + 2$  ( $c$  currency coefficients, intercept, and variance). This leads to the same estimating functions (up to scaling) and estimates compared to OLS for the coefficients  $\beta$  but adds another estimating function for the error variance:

$$\psi_\beta(y, x, \beta, \sigma^2) = x(y - x^\top \beta)/\sigma^2, \quad (19)$$

$$\psi_{\sigma^2}(y, x, \beta, \sigma^2) = ((y - x^\top \beta)^2 - \sigma^2)/(2\sigma^4). \quad (20)$$

The combined estimating functions  $\psi$  are employed for testing and monitoring exchange rate regimes and  $\Psi_{\text{NLL}}$  is used for dating breaks between the regimes. Note that breaks (rather than smooth transitions) are particularly likely to be a useful model here because changes in the exchange rate regime typically stem from policy interventions of the corresponding central banks.

### 3.2. China

In recent years, there has been enormous global interest in the CNY exchange rate which was fixed to the USD in the years leading up to mid-2005. In July 2005, China announced a small appreciation of CNY, and, in addition, a reform of the exchange rate regime. The People's Bank of China (PBC) announced this reform to involve a shift away from the fixed exchange rate to a basket of currencies with greater flexibility (PBC 2005).

Despite the announcements of the PBC, little evidence could be found for China moving away from a USD peg in the months after July 2005 (Shah, Zeileis, and Patnaik 2005). To begin our investigation here, we follow up on our own analysis from autumn 2005: Using daily returns from 2005-07-26 up to 2005-10-31 (corresponding to  $n = 68$ ), we established a stable exchange regression in Shah *et al.* (2005) that we monitored in the subsequent months, publishing the monitoring progress weekly online at [http://www.mayin.org/ajayshah/papers/CNY\\_regime/](http://www.mayin.org/ajayshah/papers/CNY_regime/). The currency basket employed consists of the most important floating currencies: USD, JPY (Japanese yen), EUR (euro), and GBP (British pound). More currencies could be included but we refrain to do so because including irrelevant currencies decreases power and precision of the procedures and the most important conclusions can already be drawn with this small basket, as we will see below.

In a first step, we fit the exchange regression to the 68 observations in the first three months after the announcements of the PBC. The estimated exchange rate regime is

$$\text{CNY}_i = -0.005 + 0.9997 \text{USD}_i + 0.005 \text{JPY}_i - 0.014 \text{EUR}_i - 0.008 \text{GBP}_i + \hat{u}_i.$$

with only the USD coefficient differing significantly from 0 (but not significantly from 1), thus signalling a very clear USD peg. The  $R^2$  of the regression is 99.8% due to the extremely low standard deviation of  $\sigma = 0.028$ .

The fluctuation in the parameters during this history period is very small, see the corresponding  $efp(t)$  for  $0 \leq t \leq 1$  in Figure 1 on the left of the vertical dashed line (marking the end of the history period). Also none of the parameter instability tests from Section 2.2 would reject the null hypothesis of stability, e.g., the double maximum statistic is  $S_{dmax} = 1.097$  with a  $p$  value of  $p = 0.697$ .

The same fluctuation process  $efp(t)$  is continued subsequently as described in Section 2.3 in the monitoring period starting from 2005-11-01 as shown in Figure 1 on the right of the vertical dashed line. The boundary shown is  $b(t) = \pm 2.475 \cdot t$ , derived at 5% significance level (for monitoring up to  $T = 4$ ). In the first months, up to spring 2006, there is still moderate fluctuation in all processes signalling no departure from the previously established USD peg. In fact, the only larger deviation during that time period is surprisingly a *decrease* in the variance—corresponding to a somewhat tighter USD peg—which almost leads to a boundary crossing in January 2006. However, the situation relaxes a bit before in March 2006 the variance component of the fluctuation process starts to deviate clearly from its zero mean. This corresponds to an increase in the variance and leads to a boundary crossing in 2006-03-27. The fluctuation in all other coefficients remains non-significant which conveys that a USD peg is still in operation, only with a somewhat larger variance.

To capture the changes in China's exchange rate regime more formally, we fit a segmented exchange rate regression by dating regime changes as described in Section 2.4. Using daily returns from 2005-07-26 through to 2007-12-31, we determine the optimal breakpoints for  $m = 1, \dots, 10$  breaks with a minimal segment size of  $n_h = 20$  observations and compute

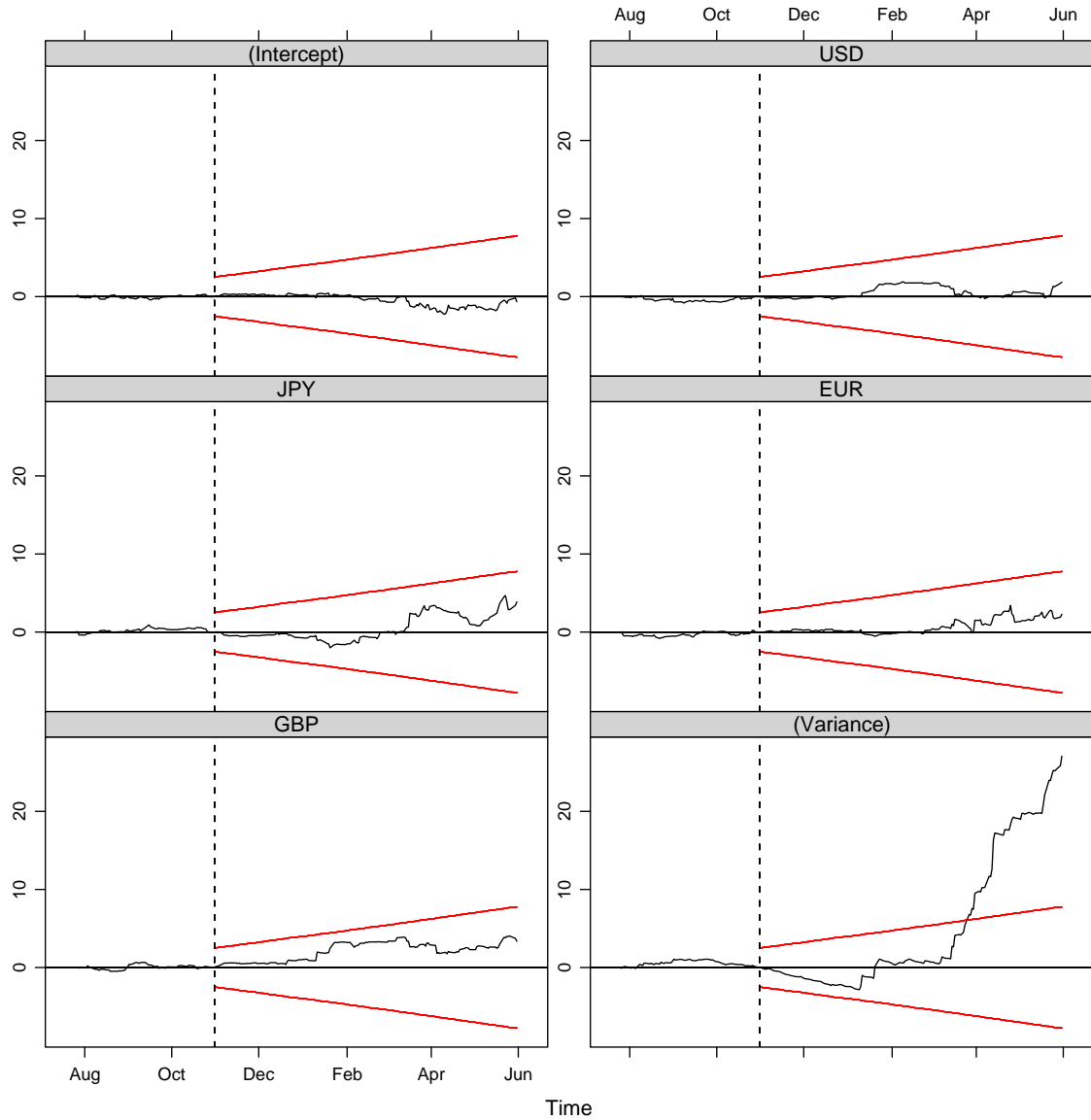


Figure 1: Monitoring fluctuation process for CNY exchange rate regime.

the associated segmented NLL and LWZ criterion. Of course, NLL decreases with every additional break but with a marked decrease only for going from  $m = 0$  to 1 break. This is also reflected in the LWZ criterion that assumes its minimum for  $m = 1$  so that we choose a 1-break (or 2-segment) model. The estimated breakpoint is 2006-03-14, i.e., shortly before the boundary crossing in the monitoring procedure (which occurs somewhat later due to the time the process needs to deviate from zero to the boundary). The associated parameter estimates are provided in Table 1 along with standard errors. The 90% confidence interval for the break date estimate is [2006-02-17, 2006-03-15], which is rather non-symmetric. Roughly speaking, this means that the end of the low-variance regime can be determined more precisely than

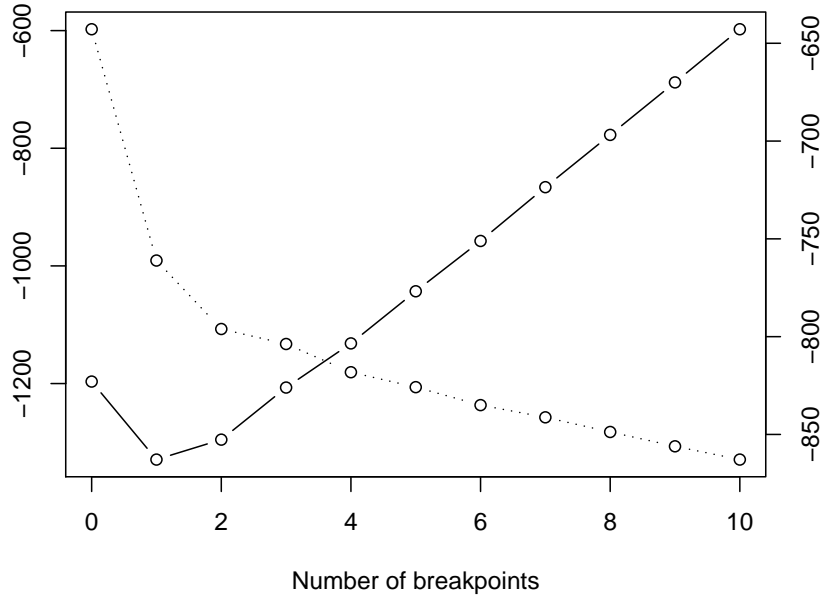


Figure 2: Negative log-likelihood (dotted, left axis) and LWZ information criterion (solid, right axis) for CNY exchange rate regimes.

the start of the high-variance regime.

These results allow for several conclusions about the Chinese exchange rate regime after spring 2006: CNY is still pegged to USD. The exchange rate regime got more flexible, going from  $\sigma = 0.028$  to 0.096. While the change is statistically significant, this is still very low (see results below for India). The intercept is clearly smaller than 0, reflecting a slow appreciation of the CNY. In summary, there is a modest easing of the rigid USD peg in spring 2006. To assess whether the CNY regime makes further steps away from tight pegging to USD, a new monitoring process could be set up using the data after 2006-03-14 as the history period (or potentially bounding the start somewhat away from the break).

period	$\beta_0$	$\beta_{\text{USD}}$	$\beta_{\text{JPY}}$	$\beta_{\text{EUR}}$	$\beta_{\text{GBP}}$	$\sigma$
2005-07-26 – 2006-03-14	<b>-0.005</b> (0.002)	<b>0.999</b> (0.005)	0.005 (0.005)	-0.015 (0.017)	0.007 (0.008)	0.028
2006-03-15 – 2007-12-31	<b>-0.020</b> (0.005)	<b>0.977</b> (0.013)	-0.015 (0.011)	0.030 (0.029)	-0.009 (0.016)	0.096

Table 1: Segmented CNY exchange rate regimes: Coefficient estimates (and standard errors) with significant coefficients (at 5% level) printed in bold face.

### 3.3. India

Another expanding economy with a currency that has been receiving increased interest over the last years is India. As with China, India is in the process of evolving away from a closed economy towards a greater integration with the world on both the current account and the capital account. This has brought considerable stress upon the pegged exchange rate regime. Therefore, we try to track the evolution of the INR exchange rate regime since trading in the INR began. Using weekly returns from 1993-04-09 through to 2008-01-04 (yielding  $n = 770$  observations), we first fit a single exchange regime that is subsequently segmented. Weekly

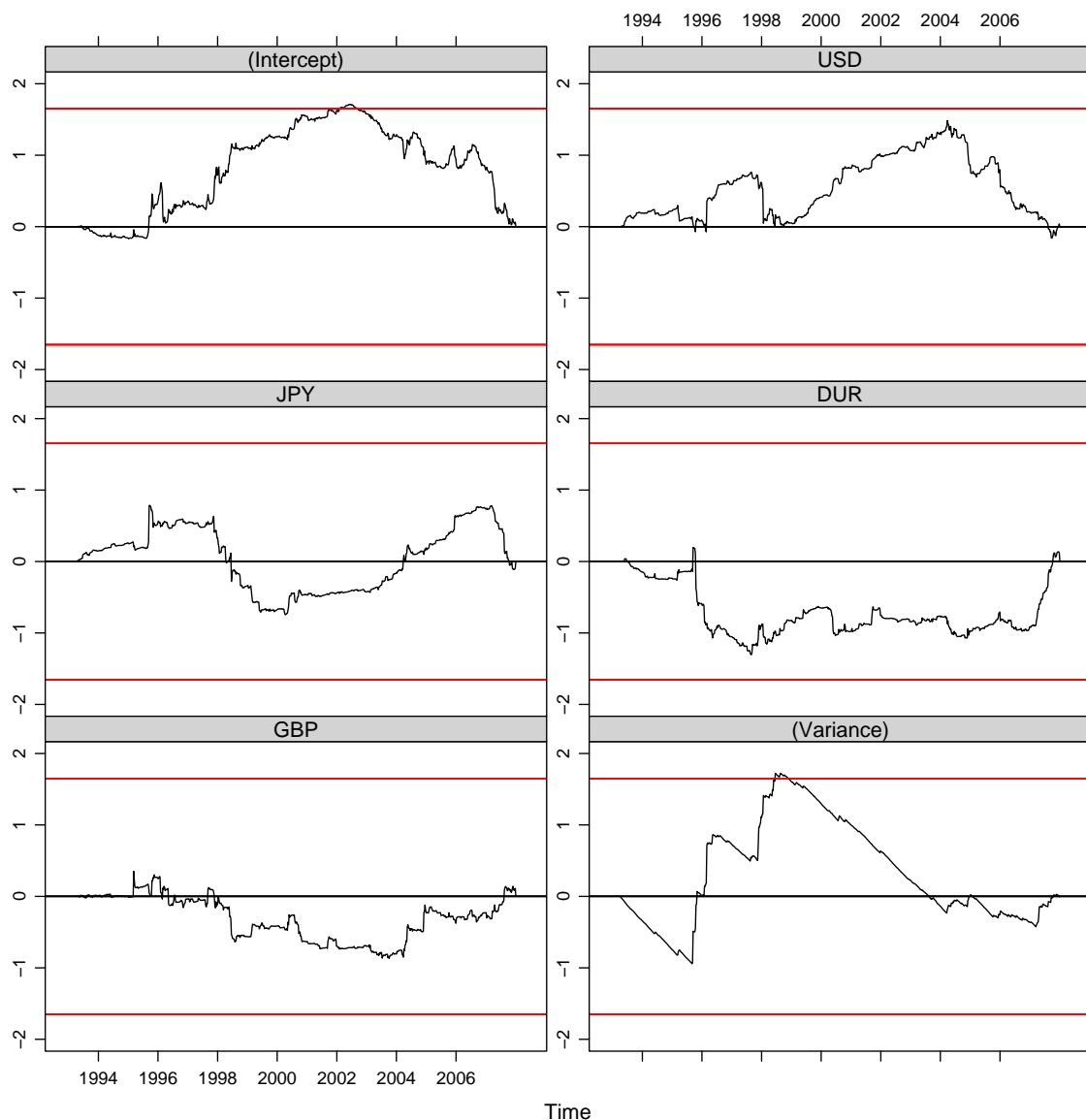


Figure 3: Historical fluctuation process for INR exchange rate regime.

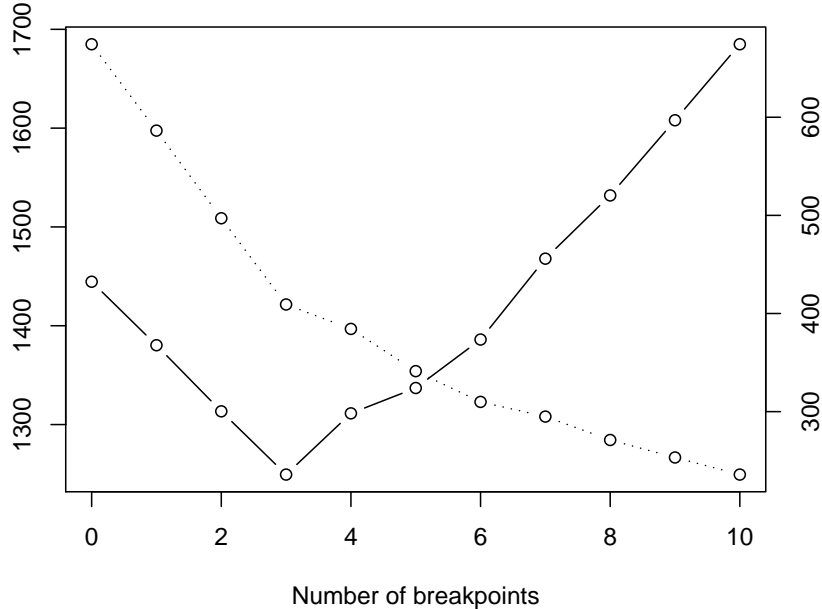


Figure 4: Negative log-likelihood (dotted, left axis) and LWZ information criterion (solid, right axis) for INR exchange rate regimes.

rather than daily returns are employed to reduce the noise in the data and alleviate the computational burden of the dating algorithm of order  $O(n^2)$ . The currency basket employed is essentially the same as above using the most important floating currencies.<sup>1</sup>

Using the full sample, we establish a single exchange rate regression only to show that there is not a single stable regime and to gain some exploratory insights from the associated fluctuation process. As we do not expect to be able to draw valid conclusions from the coefficients of a single regression, we do not report the coefficients here and rather move on to a visualization of  $efp(t)$  and the associated double maximum test in Figure 3. Because two processes (intercept and variance) exceed their 5% level boundaries, there is evidence for at least one structural change. More formally, the test statistic is  $S_{dmax} = 1.724$  with a  $p$  value of  $p = 0.031$ . This  $p$  value is not very small because there seem to be several changes in various parameters. A more suitable test in such a situation would be the Nyblom-Hansen test with  $S_{CvM} = 3.115$  and  $p < 0.005$ . Nevertheless, the multivariate fluctuation process is interesting as a visualization of the changes in the different parameters. The process for the variance  $\sigma^2$  has the most distinctive shape revealing at least four different regimes: at first, a variance that is lower than the overall average (and hence a decreasing process), then a much larger variance (up to the boundary crossing), a much smaller variance again and finally a period where the

<sup>1</sup>The only difference to the previous section is that EUR can only be used for the time after its introduction as official euro-zone currency in 1999. For the time before, exchange rates of the German mark (DEM, the most important currency in the EUR zone) adjusted to EUR rates, are employed. The combined returns are denoted DUR below.

period	$\beta_0$	$\beta_{\text{USD}}$	$\beta_{\text{JPY}}$	$\beta_{\text{DUR}}$	$\beta_{\text{GBP}}$	$\sigma$
1993-04-09 – 1995-03-03	-0.006 (0.017)	<b>0.972</b> (0.018)	0.023 (0.014)	0.011 (0.032)	0.020 (0.024)	0.157
1995-03-10 – 1998-08-21	<b>0.161</b> (0.071)	<b>0.943</b> (0.074)	0.067 (0.048)	-0.026 (0.155)	0.042 (0.080)	0.924
1998-08-28 – 2004-03-19	0.019 (0.016)	<b>0.993</b> (0.016)	0.010 (0.010)	<b>0.098</b> (0.034)	-0.003 (0.021)	0.275
2004-03-26 – 2008-01-04	-0.058 (0.042)	<b>0.746</b> (0.045)	<b>0.126</b> (0.042)	<b>0.435</b> (0.116)	<b>0.121</b> (0.056)	0.579

Table 2: Segmented INR exchange rate regimes: Coefficient estimates (and standard errors) with significant coefficients (at 5% level) printed in bold face.

variance is roughly the full-sample average. Other interesting processes are the intercept and maybe the USD and DUR. The latter two are not significant but have some peaks revealing a decrease and increase, respectively, in the corresponding coefficients.

To capture this exploratory assessment in a formal way, a dating procedure is conducted for  $m = 1, \dots, 10$  breaks and a minimal segment size of  $n_h = 20$  observations. The resulting values for the NLL and associated LWZ information criterion are depicted in Figure 4. NLL is decreasing quickly up to  $m = 3$  breaks with a kink in the slope afterwards. Similarly, LWZ takes its minimum for  $m = 3$  breaks, choosing a 4-segment model. The corresponding parameter estimates (and standard errors) are reported in Table 2.

The most striking observation from the segmented coefficients is that INR was closely pegged to USD up to March 2004 when it shifted to a basket peg in which USD has still the highest weight but considerably less than before. Furthermore, the changes in  $\sigma$  are remarkable, roughly matching the exploratory observations from the empirical fluctuation process. A more detailed look at the results in Table 2 shows that the first period is a clear and tight USD peg. During that time, pressure to appreciate was blocked by purchases of USD by the central bank. The second period, including the time of the East Asian crisis, saw a highly increased flexibility of the INR. The third period exposes much tighter pegging again with low volatility, some appreciation and some small (but significant) weight on DUR. In the fourth period after March 2004, India moved away from the tight USD peg to a basket peg involving several currencies with greater flexibility (but smaller than in the second period). In this period, reserves in excess of 20% of GDP were held by the Reserve Bank of India (RBI), and a modest pace of reserves accumulation has continued.

The confidence intervals for the three break dates at 90% level are [1994-11-11, 1995-03-10], [1998-08-14, 1998-12-18], and [2003-11-28, 2004-04-02], respectively. These are again rather non-symmetric with tight bounds corresponding to the low-variance regimes. (Recall that weekly data is employed and thus a difference of seven days is the tightest bound possible.)

The existing literature classifies the INR is a *de facto* pegged exchange rate to the USD in the period after April 1993 (Reinhart and Rogoff 2004). Table 2 shows the fine structure of this pegged exchange rate; it supplies dates demarcating the four phases of the exchange rate regime; and it finds that by the fourth period, there was a basket peg in operation. This constitutes a statistically well-founded alternative to the existing classification schemes of the Indian exchange rate regime.

## 4. Summary

A unified structural change toolbox for testing, monitoring, and dating structural changes in maximum likelihood regression models is discussed. All techniques are based on the (negative) log-likelihood as the objective function or the corresponding estimating function (likelihood scores), respectively. In particular, methods for estimating the breakpoints in such a model are extended from the corresponding techniques for least-squares regression. To illustrate their usefulness in practice, all procedures are applied to a regression model for estimating exchange rate regimes. Specifically, changes in the regimes of two currencies are investigated: CNY and INR. For the former, a 2-segment model is found for the time after July 2005 when China gave up on a fixed exchange rate to the USD. While being still closely linked to USD in both periods, there has been a small step in the direction of the claims of the Chinese central bank: flexibility slightly increased. For the INR, a 4-segment model is found with a close linkage of INR to USD in the first three periods (with tight/flexible/tight pegging, respectively) before moving to a more flexible basket peg in spring 2004.

## Computational details

The cross-currency returns are derived from exchange rates available online from the US Federal Reserve at <http://www.federalreserve.gov/releases/h10/Hist/>. All computations are carried out in the R system for statistical computing (R Development Core Team 2008, version 2.7.1) with packages **fxregime** 0.2-0 (Zeileis, Shah, and Patnaik 2008) and **strucchange** 1.3-3 (Zeileis, Leisch, Hornik, and Kleiber 2002). Both, R itself and the packages, are freely available at no cost under the terms of the GNU General Public Licence (GPL) from the Comprehensive R Archive Network at <http://CRAN.R-project.org/>. Vignettes reproducing the analyses from this paper are available via `vignette("CNY", package = "fxregime")` and `vignette("INR", package = "fxregime")` after installing the packages.

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**Affiliation:**

Achim Zeileis  
Department of Statistics and Mathematics  
Wirtschaftsuniversität Wien  
Augasse 2–6  
AT-1090 Wien, Austria  
E-mail: [Achim.Zeileis@R-project.org](mailto:Achim.Zeileis@R-project.org)  
URL: <http://statmath.wu-wien.ac.at/~zeileis/>

Ajay Shah, Ila Patnaik  
National Institute of Public Finance and Policy  
18/2 Satsang Vihar Marg  
New Delhi 110067, India  
E-mail: [AjayShah@mayin.org](mailto:AjayShah@mayin.org), [IlaPatnaik@gmail.com](mailto:IlaPatnaik@gmail.com)  
URL: <http://www.mayin.org/ajayshah/>, <http://openlib.org/home/ila/>